Weighing the Earth using Arthur's Seat

In the 18th century what lay beneath the Earth's surface was hotly debated. Was the Earth hollow or solid? Was its interior full of ocean waters that entered through deep chasms, filled with fermenting mud, or were Earth's subterranean realms inhabited by strange creatures and spirits, as in the Celtic mythology of "Cruachan"? A pioneering C19th geophysical experiment used Arthur's Seat in a mathematical quest to successfully determine the Earth's mass and to prove Earth had a dense, solid internal structure.

Background

Following the Copernican Revolution two fundamental parameters (the mass of the Earth and the Sun-Earth distance) were needed for establishing our place in the universe.

The Earth mass is a standard unit of mass in astronomy. It is used as the reference for all the other planets. The Sun-Earth distance is the well-known astronomical unit (1 AU), and is still used for measuring distances within the Solar System and also the large distances to astronomical objects outside the Solar System (in terms of the parsec),

Precise measurement of the Earth mass is particularly difficult (it is equivalent to measuring the gravitational constant, which is the least well known of all the fundamental physical constants). Earth's mass was first measured reasonably well in the Schiehallion mountain experiment of 1774 (which resulted in a value about 20% too low).

Using a Mountain to Weigh the World

The basic idea is to compare the relative masses of a mountain (in this case Arthur's Seat) and the Earth by measuring the deflection of a plumb line as caused by the mountain's (sideways) gravitational pull in comparison to the Earth's (downward) pull.

The deflections are tiny*. Even in the Himalayas, mountain peaks only generate deviations of up to 0.03° (~100″). Such minute departures, from the vertical, must be ascertained relative to the fixed background of the stars using a series of extremely careful observations.

The mass of a mountain must be obtained from its volume and the density of its rocks.

Instrumentation

Absolutely critical to the success of the Arthur's Seat experiment were the observing instruments. The main observing equipment used by the Ordinance Survey, who performed the Arthur's Seat investigation, were large theodolites and a Zenith Sector. Airy's portable Zenith Sector proved to be particularly valuable. It was a new type of telescope designed, in effect, to point straight up and make precise measurements of star positions (Fig. 1). While a substantial instrument, it employed an extremely sensitive levelling system and possessed an eyepiece fitted with a micrometer microscope for measuring very small angles.

Because the sole purpose of Airy's Zenith Sector was to determine the latitude of the observing station with the greatest possible accuracy, restricting observations to near the vertical was a cleaver ploy. Errors arising from tube flexure were much reduced, as was refraction of starlight because it only passed vertically through the Earth's atmosphere.

The Arthur's Seat observing Stations were established where the large theodolites commanded a good view, allowing a connection by triangulation into the main (countrywide) National Grid (Fig. 2) and hence providing a precise (within inches) geographical (latitudinal) location.

Once a Station's position was well established it became possible, from the difference in location of a star vs. its predicted location to determine the local deflection of the vertical.

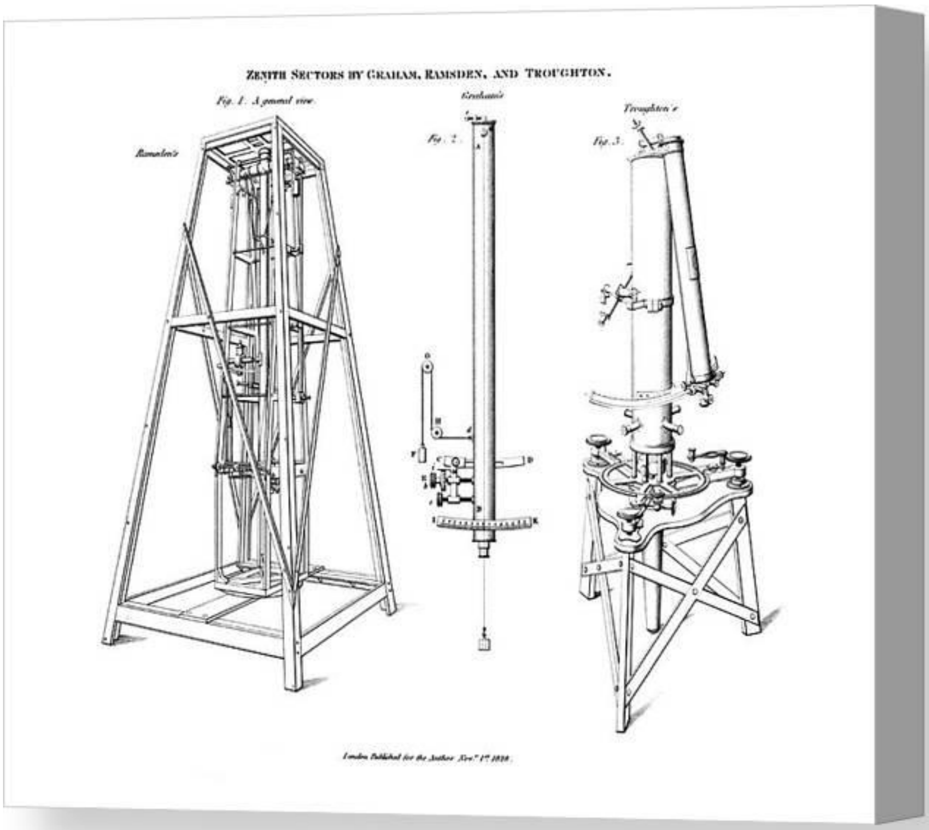

Fig. 1. Example of Airy's portable instument. From "An introduction to practical astronomy" by Pearson, 1828.

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The Arthur's Seat Survey

Lieutenant-Colonel Henry James published the results of the Ordinance Survey's study on Feb. 7, 1856. *https://royalsocietypublishing.org/doi/pdf/10.1098/rstl.1856.0029* Having obtained 2.75 g/cm^3, for the mean density of Arthur's Seat, he arrived (by comparing the gravitational force from the Earth pulling "down" with the gravitational force from the mountain pulling "sideways") at a mean density of Earth of 5.316. In retrospect, a remarkably good outcome.

First, James deduced the mean density and volume of Arthur’s Seat. As the geological structure of Arthur’s Seat is well exposed, its effective density (2.75 g/cm^3) could be derived from measurements on its ten rock types. Its volume and shape were found from a contoured plan made, at the scale of 6 inches to a mile, while surveying Edinburghshire.

Secondly, in terms of the deflection of the vertical, it is prudent to focus on the gravitational pull only in the north-south direction, as all measurements are then much easier. Accordingly, observation Stations were positioned on the north and south flanks of Arthur’s Seat, as well as close to the summit (strictly, 14 ft. from the Triangulation Pillar, at bearing 18° NW). It is also beneficial to position the flank Stations at about one-third of altitude above the surrounding country (in order to be at a similar elevation to the mountain’s centre of mass and so maximize the horizontal pull). The North Station (Figs. 3, 5, 6, 7) was placed at a distance of 2490.0 ft., and the South Station (Figs. 4, 8) 1426.7 ft., from the summit of Arthur’s Seat and close to the meridian line through the peak (Fig. 4).

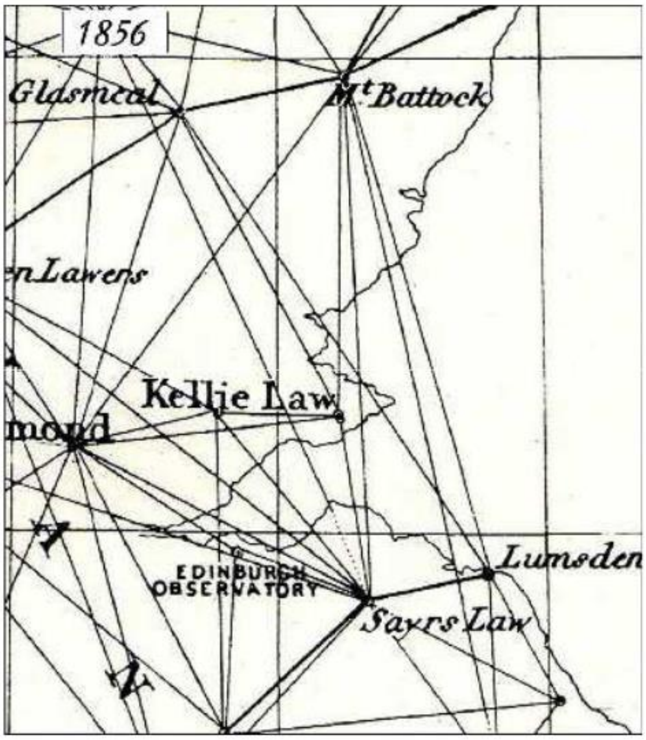


Fig. 2. Extract from a map showing the Ordnance Survey retriangulation of Great Britain, into which the Arthur’s Seat Zenith Stations were linked. In this way the summit Station was determined to lie at a geodetic latitude of 55° 56' 38" 31.

https://www.charlesclosesociety.org/files/Issue98page5.pdf

The gravitational pull at all three Stations was found, by the Survey’s astronomical sightings, to be deflected to the south by nearly equal amounts of 5" 25. Examination of regional topographic and bathymetric maps (out to distances as far as the border of the county of Peebles) revealed the attraction by the distant Pentland Hills to be the dominant cause (some 4" 88) of the southward (5" 25) tilt. In contrast, the water filled hollow of the Forth Estuary, to the north, was found to account for but a small portion of the southward tilt.

More locally, the attraction of the mass of Arthur’s Seat was determined by the differences between the North and South Zenith Stations. After corrections, a North Station tilt of 2" 00 towards Arthurs Seat was found, while to the South a 2” 21 tilt northwards (once again inwards towards Arthurs Seat) was deduced. This 4” 21 divergence is James’s key finding.

Summary

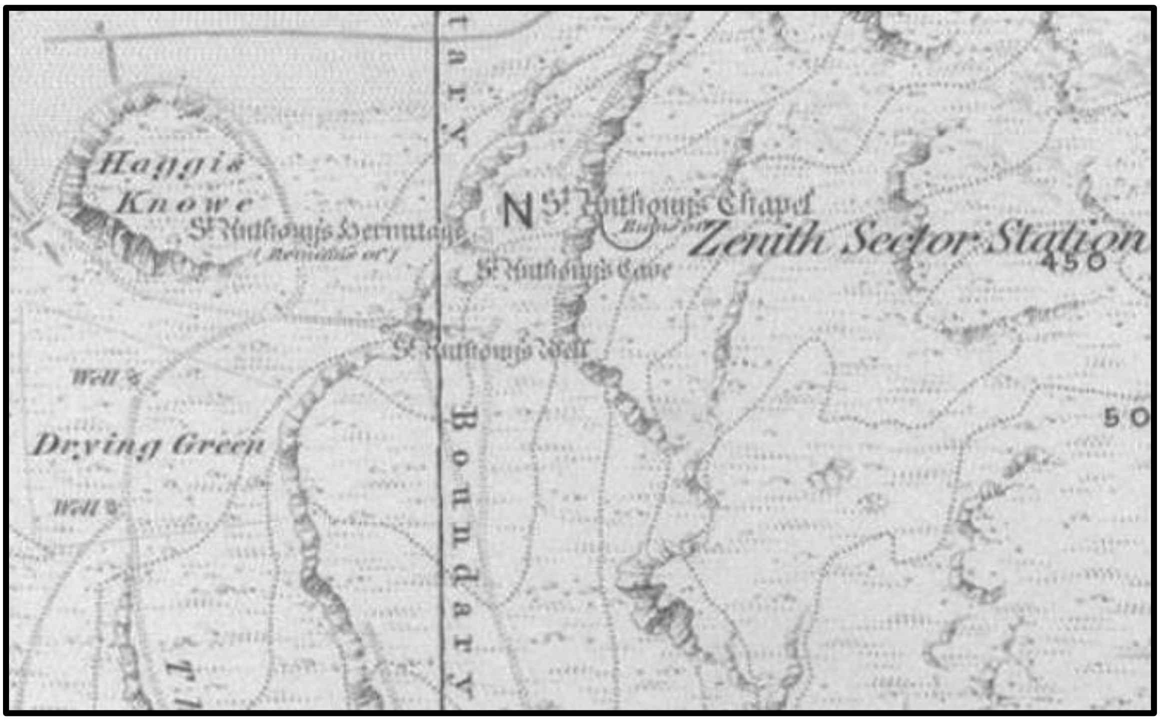
When, in 1856, the astronomically derived tilts observed adjacent to Arthur’s Seat were combined with new measurements of the hill’s density and volume, they yielded a mass for the Earth of 13,000,000,000,000,000,000,000,000 pounds. This historically important result is remarkably close to today’s accepted value (as based on Cavendish-type experiments).

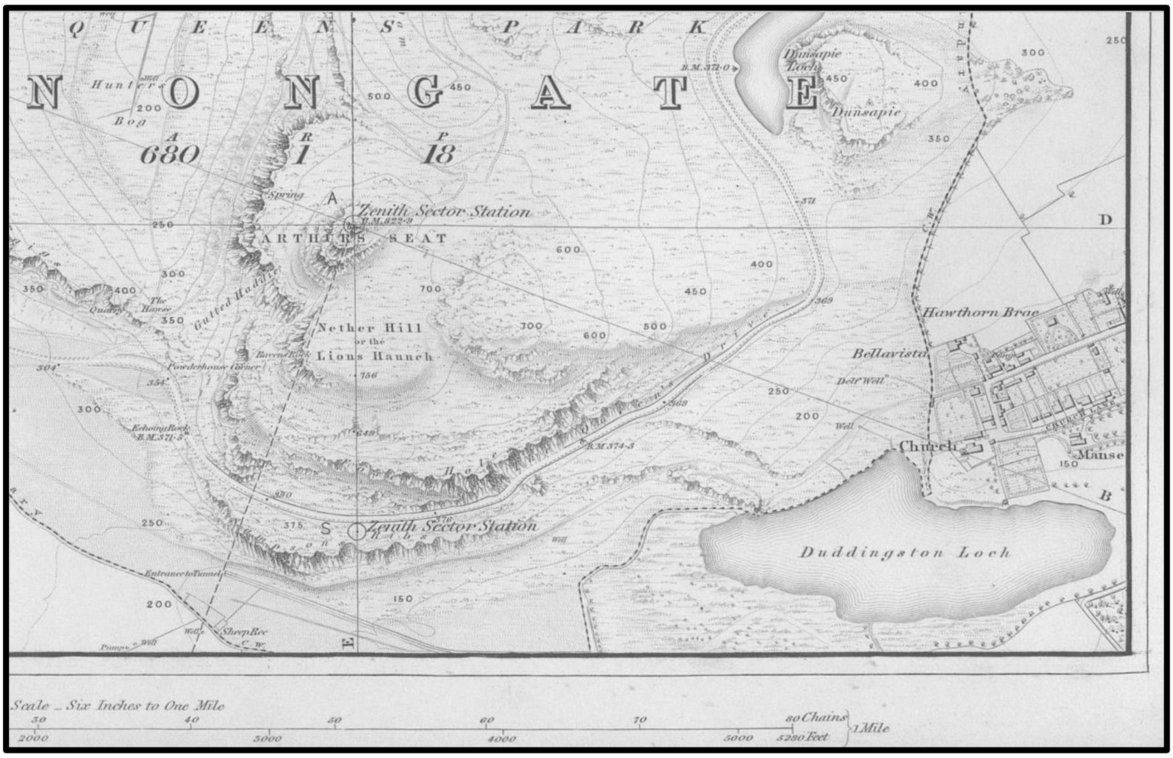
** It may be helpful to imagine the tiny angular deflections under observation in terms of a perturbation in the level of the nearby (Olympic sized) Commonwealth Swimming*

Pool. One second of arc (1") corresponds to a tilt in water surface, across the pool's length, by the width of a pinhead. As a measure of progress, modern (electronic) Zenith cameras can deliver vertical deflections accurate to about 0.1 second of arc.

Self-guided walk

A walking route around Arthur's Seat which visits the sites of the former North and South Zenith Stations is described at: https://blogs.ed.ac.uk/roythompson/wp-content/uploads/sites/566/2020/12/ArthursSeat.pdf **RT Aug 2021**





Figs. 3 & 4. North and South Zenith Station maps modified from "On the Deflection of the Plumb-Line at Arthur's Seat, and the Mean Specific Gravity of the Earth", James, Phil. Trans. Roy. Soc. London Vol. 146 (1856), pp. 591-606.

Fig. 5. The North Zenith Station sat in the protected hollow to the left (east) of the crags immediately above St Anthony's Chapel.

Fig. 6. Site of North Zenith Station (to the left of the walking pole) with good view to the summit of Arthur's Seat. NB Distant Pentland Hills (just visible, as a dark grey ridge, through the col) affect the vertical even more (4.88") than the mass of Arthur's Seat (2.00").

Fig. 7. View from the North Zenith Station, looking down the crags to ruined Chapel and loch below. A Zenith Station was also established alongside the key triangulation locality at the Royal Edinburgh Observatory on Calton Hill (adjacent to Nelson's column on skyline). The distant hills lie across the Forth Estuary in Fife.

Fig. 8. Site of South Zenith Station (behind the walking pole handle) with Arthur's Seat above and to the left. Extensive views lead eastwards across Duddingston Loch to broaden out to the distant Lammermuir Hills (including the principal triangulation point on their highest peak of Meikle Says Law).