Energy transfer and dissipation in isotropic turbulence: a unified, quantitative treatment of the energy cascade.

This may seem a rather grandiose title for a blog, even if it is the first blog of the year! However, it is in fact the title of my new book, for which I have a contractual deadline of 30th of September. In other words, not much time to post blogs!

Despite this, I still hope to post the occasional short blog on matters that arise while I am writing the book. For instance, there is the use of the word ‘unified’ in the title. What exactly does this mean? Well, it means quite simply giving parallel treatments of various matters in both physical space and wave number space.

Somewhat surprisingly, when my book proposal was sent out for review, all four anonymous reviewers welcomed this particular aspect. I say surprisingly because of the many people who study turbulence a large proportion prefers to work in physical space. Indeed, their reactions to the use of wave number space can range from distaste and incomprehension to outright hostility: see my blog of 20th February 2020 for a discussion of this.

Of course the virtues of using integral transforms to replace differentials by multipliers are well known. But, as one of the reviewers pointed out, the use of Fourier transformation with respect to wave number is necessary in order to reveal, and to exploit, underlying symmetries.

An important example of this arises from a comparison of Kolmogorov’s 1941 theory in physical space with Onsager’s 1947 theory in wave number space. The first of these, despite its great importance, is rather imprecise and contains internal inconsistencies. In contrast Onsager’s theory relies on scale invariance and is more quantitative in nature. This is something that will receive a fuller treatment in the book.