Onsager’s (1945) interpretation of Kolmogorov’s (1941a) theory: 1

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It is a well known fact that as one gets older time speeds up! This (apparent) reverse
time-dilation is something to be reckoned with once one has passed `the big eight-oh’. So much so, that I was convinced that I had posted blogs quite recently only to find that my last post was in January of this year and that was my only post so far this year. Obviously I have my excuses, such as involvement with a special issue of the journal Atmosphere, as mentioned in a previous post. In particular I have been preparing some potential editorial material which I have posted on the physics ArXiV, while I consider what to do with it [1]. However, that work has stimulated various lines of thought which have led to the present post. Or rather, what I expect to be a series of posts.

My thinking on this goes back to 1999, when I was a participant in the turbulence programme at the Isaac Newton Institute. At one of our informal meetings, there was some discussion of the fact that Kolmogorov had not actually derived the eponymous -5/3 spectrum, but in fact the 2/3 power law for the second-order structure function [2]. This may seem a strange observation to make; but, from the 1950s onwards, theorists had worked exclusively in terms of wavenumber space, and the test of a successful theory was its ability to reproduce the Kolmogorov spectrum. By this stage there was a growing awareness that Kolmogorov had not actually derived the

distribution that bears his name. Someone then asked who had derived the -5/3 spectrum. It seemed that no one was quite sure, but there was general agreement that it must have been Obukhov, and there the matter rested.

Nevertheless, this did not seem an entirely satisfactory answer, because Obukhov's paper [3], although in spectral space, introduced a turbulent effective viscosity, in order to calculate the inertial-range spectrum. So that was something that I wondered about. Then it turned out that the answer was hiding in plain sight all the time.

In 1945, Onsager published an abstract [4], giving a brief summary of his method, which identified the key importance the inertial flux of energy through wavenumber. In 1949, he published a fuller account of his method, in which he introduced the idea of the energy transfer being a cascade [5]. In 1964, Corrsin published an exegesis of this work, and even used the term `Onsager's model' in his title [6]. A noteworthy feature of this work is that it relied on the introduction of a modal decay time, which was determined, like the energy spectrum, by dimensional analysis. This is of interest because of its relationship to the renormalization methods being introduced by Kraichnan [7] and Edwards [8] at much the same time.

We shall return to this point in later posts. For the moment, we note that the work of Onsager was brought to a wider audience by the 2006 review of Eyink and Sreenivasan [9], which also pointed out that Onsager had introduced the term `energy cascade', as of course we all use today.

References

[1] W. D. McComb. What is isotropic turbulence and why is it important?
arXiv:2403.13962v1[math-ph], 2024.
[2] A. N. Kolmogorov. The local structure of turbulence in

incompressible viscous fluid for very large Reynolds numbers. C. R. Acad. Sci. URSS, 30:301, 1941.
[3] A. M. Obukhov. On the energy distribution in the spectrum of turbulent flow. C.R. Acad. Sci. U.R.S.S, 32:19, 1941.
[4] L. Onsager. The Distribution of Energy in Turbulence. Phys. Rev., 68:286, 1945.
[5] L. Onsager. Statistical Hydrodynamics. Nuovo Cim. Suppl., 6:279, 1949.
[6] S. Corrsin. Further Generalization of Onsager's Cascade Model for Turbulent Spectra. Phys. Fluids, 7:1156{1159, 1964.
[7] R. H. Kraichnan. The structure of isotropic turbulence at very high Reynolds numbers. J. Fluid Mech., 5:497{543, 1959.
[8] S. F. Edwards. The statistical dynamics of homogeneous turbulence. J. Fluid Mech., 18:239, 1964.
[9] G. L. Eyink and K. R. Sreenivasan. Onsager and the Theory of Hydrodynamic Turbulence. Rev. Mod. Phys., 87:78, 2006.