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There has been an increasing awareness in the turbulence community of the significance of finite-Reynolds-number (FRN) effects, corresponding to an impression that Kolmogorov's theory requires an infinite Reynolds number. At the same time, there has been a recent growth of interest in *Onsager's Conjecture*, which is essentially taken to mean that turbulent dissipation is still present even when the Reynolds number is infinite, and this is normally interpreted as being when the fluid viscosity is zero. Oddly, there never seems to be any mention of the word '*limit*'; and one detects a degree of uncertainty about the whole matter, with typical comments like: 'when the Reynolds number is infinite, or at least very large'.

In order to examine this topic, we may begin by remarking that the Newtonian fluids we study always have a finite viscosity. Also, reducing the viscosity seems like an unlikely method of increasing the Reynolds number. If we take pipe flow as an example, the normal procedure is to increase the velocity of the fluid, as being much easier than increasing the pipe diameter or decreasing the fluid viscosity. Nevertheless, the idea of varying the viscosity has been around for a long time, with Batchelor discussing the idea of taking the limit as $\nu \rightarrow 0$, at a constant rate of dissipation. He argued that this would push the effect of viscosity to an infinite value of the wavenumber, i.e. $k=\infty$ [1]. Edwards took the idea further; and, in order to test his statistical theory, argued that the input (due to forces) and the output (due to viscosity) could be represented by delta functions at $k=0$ and $k=\infty$, respectively [2]. However, both examples were in the context of continuum mechanics and, most importantly, involved the taking of limits. That is, the case of $\nu=0$ is the Euler equation, and there is no dissipation. The case $\nu \rightarrow 0$, such that dissipation is maintained constant, involves a limiting process and is the *Navier-Stokes equation in the infinite Reynolds limit*. It is not the Euler equation.

This procedure of taking the viscosity to tend to zero can seem counter-intuitive; but an example that is even more so can be found in microscopic physics, where the classical limit can be obtained by letting Planck’s constant h tend to zero. This is definitely counter-intuitive: after all, h is not just a constant, it is a universal constant. The answer to this is that we must be taking a limit where the quantum levels become infinitesimally small in comparison to the energies involved in the macroscopic system.

We can apply the same idea to turbulence. For pipe flow, we can work with a non-dimensional viscosity of the form $\tilde{\nu}=\nu/U d$, where U is the bulk mean velocity and d is the diameter of the pipe. Evidently increasing the velocity is the equivalent of decreasing the scaled viscosity. Moreover, as the scaled velocity is a pure number, concepts of large and small are better defined.

The idea of zero scaled viscosity then corresponds to an infinite value of the velocity and clearly is not achievable. So, in practice, zero scaled viscosity means that it is small enough compared to other relevant quantities that it may be neglected. The best way to do this, is to look for a limit. That is, as the scaled viscosity tends to zero, the dissipation (say) tends asymptotically to a constant value. When variations in the dissipation are too small to resolve, either numerically or experimentally, we have in fact reached a limiting value. This behaviour can be seen in the variation of dissipation rate with increasing Reynolds number in reference [3].

[1] G. K. Batchelor. The theory of homogeneous turbulence.

Cambridge University Press, Cambridge, 2nd edition, 1971. (First published 1953).
[2] S. F. Edwards. Turbulence in hydrodynamics and plasma physics. In Proc. Int. Conf. on Plasma Physics, Trieste, page 595. IAEA, 1965.
[3] W. D. McComb, A. Berera, S. R. Yoffe, and M. F. Linkmann. Energy transfer and dissipation in forced isotropic turbulence. Phys. Rev. E, 91:043013, 2015.

Postscript. This is my first post in some time because I have been busy with acting as Guest Editor in a special edition of Atmosphere. I hope to post frequently from now on. The link to the journal is:
https://www.mdpi.com/journal/atmosphere/special_issues/FOH7AK5UB1