Two-time correlations and temporal spectra: the Lagrangian case.

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In my previous post on 27 April 2023, I promised to come back to the Lagrangian case. Over the years, I have taken the view that the discussion of the Lagrangian case along with the Eulerian case, which is the one that is of more practical importance, is an unnecessary complication. At the same time, I have had to acknowledge that the application of these ideas to the assessment of statistical closure theories should take account of the fact that there are Lagrangian theories as well as Eulerian theories. However, there is an interesting point to be made when we compare the treatment in the book by Tennekes and Lumley [1] with the later analysis of Tennekes [2].

In the previous post, we only mentioned the discussion by Tennekes and Lumley [1] of the inertial range behaviour of the Eulerian spectrum. In fact they not only derive the inertial range form of the Lagrangian spectrum, and find it to be the same power law as the Eulerian case, but also obtain a relationship between the constants of proportionality in the two cases.

The crucial step in this work is the equivalence of the two correlations (see Section 8.5 of [1]), where the authors refer back to their discussion of Lagrangian forms in Section 7.1 (actually they incorrectly give this as 7.2). Following their notation, we represent the Lagrangian velocity of a fluid point by $V_{\alpha}(t)$ where $\alpha = 1, 2, \mbox{or}, 3$. Then, they assert that $\langle V_{\alpha}V_{\alpha}\rangle =\langle u_{\alpha} u_{\alpha}\rangle$, where u_{α} is of course the Eulerian velocity; leading on to their equation (8.5.3). This is the step that provides the basis for their assertion of the equivalence of the Eulerian and Lagrangian inertial range spectra.

However, the later work of Tennekes [2] leads to the Eulerian spectrum being different from the Lagrangian form, due to the supposed predominance of sweeping effects. This would seem to be an inconsistency and we will return to this in future posts when we examine the work of Tennekes more closely.

We close by pointing out that in our previous post we noted that the form of two-time correlation being studied in [1] was limited to stationary flows. This point was also made by Hinze [3]: see equation (1.57), page 39 in the first edition. However, in discussing the motion of fluid points in Lagrangian coordinates, Tennekes and Lumley emphasise the need for both homogeneity and stationarity. So in effect this restriction has already been made. We also note that an alternative discussion of the original work by Lumley can be found in Section 12.2 of [4].

[1] H. Tennekes and J. L. Lumley. A first course in turbulence. MIT Press. Cambridge, Mass., 1972.
[2] H. Tennekes. Eulerian and Lagrangian time microscales in isotropic turbulence. J. Fluid Mech., 87:561, 1975.

[3] J. O. Hinze. Turbulence. McGraw-Hill, New York, 1st edition, 1959.

[4] W. D. McComb. The Physics of Fluid Turbulence. Oxford University Press,
1990.