# Mode elimination: taking the phases into account: 4

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In the previous post we came to the unsurprising conclusion that as a matter of rigorous mathematics, we cannot average out the high-wavenumber modes while leaving the low-wavenumber modes unaffected. However, turbulence is a matter of physics rather than pure mathematics and the initial conditions are not known with mathematical precision. Here the concept of deterministic chaos comes to our rescue. If we accept that the initial condition must have some uncertainty attached to it, then there is a possibility that such an average can be carried out approximately.

We can generalise the conditional average, given as equation (3) in the previous post, by extending it to some arbitrary well-behaved functional $H[u(k,t)]$. Here we are also using the simplified notation of the previous post; and in fact we shall simplify it even further, and write $u(k,t)\equiv u_k$. Then we can replace that equation by:
$$\begin{equation}\langle H[u_k]\rangle_c = \langle H[u_k]\mid u^-_k \rangle,\end{equation}$$
where, as before, the subscript \`$c$' on the left hand side denotes \`conditional average'; and the notation on the right hand side indicates that the ensemble average is carried out while keeping the low-wavenumber part of the velocity field $u^-_k$ constant. From the previous discussion, we know that this average amounts to a delta function, as both $u_k$ and $u^+_k$ are also held constant.

The way out of this impasse is the recognition that, in the real physical situation, $u^-_k$ cannot be held precisely to any exact value. There must be some uncertainty, however small, in the application of this constraint. Accordingly we introduce an uncertainty into our definition of a conditional average by writing it as: \begin{equation}\langle H[u_k]\rangle_c = \langle H[u_k]\mid u^-_k + \phi^-_k\rangle.\end{equation} Evidentally, $ u^-_k + \phi^-_k$ must be a solution of the Navier-Stokes equation, but the uncertainty $\phi^-_k$ is otherwise arbitrary and may be chosen to have convenient properties. In fact, McComb, Roberts and Watt [1] chose it to satisfy the conditions:\begin{equation} \langle u^-_k\rangle_c = u^-_k + \langle \phi^-_k \rangle_c,\end{equation} along with:\begin{equation} \langle u^-_k u^-_j\rangle_c = u^-_k + \langle \phi^-_k \phi^-_j \rangle_c,\end{equation} and \begin{equation} \langle u^-_ku^+_j\rangle_c = u^-_k \langle u^+_k \rangle_c.\end{equation} These relationships are then used in decomposing the NSE and implementing an RGl calculation. It should be noted that $ \langle u^+_k \rangle_c$ is not zero and an equation of motion must be derived for it.

The problem posed by the correction terms in $\phi^-_k$ depends on just how chaotic the turbulence is, but the calculations suggest that these terms can be neglected. In fact the calculation of the invariant energy flux yields a value of the Kolmogorov spectral constant of $\alpha = 1.62$ which is the generally accepted value. Further details can be found in the original paper [1] and in the appropriate sections of the book [2].

However, despite the above procedures, there are still phase effects that are not being taken into account, and this will be the subject of the next post.

[1] W. D. McComb, W. Roberts, and A. G. Watt. Conditional-averaging procedure for problems with mode-mode coupling. Phys. Rev. A, 45(6):3507- 3515, 1992.
[2] W. David McComb. Homogeneous, Isotropic Turbulence: Phenomenology, Renormalization and Statistical Closures. Oxford University Press, 2014.