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This is my first blog this year, very largely because I have been working on a review article as my contribution to a journal issue commemorating Jack Herring, who died last year. We were asked to include some personal recollections of Jack, in addition to the physics, and I began by remarking that I had first met Jack Herring at the NASA-ICASE workshop on turbulence theory at Virginia Beach in October 1984. Taking part were physicists, mathematicians and engineers; and I, as very much a new boy, was glad to be welcomed into the physicists' group with Jack, Bob Kraichnan and various others.

This was a long time ago, and I only have a few memories of the social interactions, but I do recall that there was a `No-host cocktail hour' when the day's programme finished. I was quite amused by this phrase, which I hadn't met before, and which simply meant that the organisers weren't about to pay for the drinks! Thinking about this reminded me of an interaction with Phillip Saffman, which I think leads to a point of general interest.

My purpose at the workshop was to give a talk on my application of renormalization group (RG) to turbulence and details of that, along with the other talks may be found in the published proceedings [1]. I do not recall much discussion after the talks but I do remember that Philip Saffman stood up when questions were invited after my talk. He pointed out that my method wouldn't work because I hadn't taken the phase into account! When I joined him at the `No host cocktail hour', he said that he hoped that I didn't mind his comment. I assured him that I didn't as I had no idea what it meant. We didn't discuss it further, and spoke of other matters; but it was to act as the grit in the oyster which ultimately leads (one hopes) to a pearl.

At the time I was uneasy about my theory anyway, and began to play safe and classify it as a mean-field theory. After some years of brooding about this, and other things, I saw that eliminating modes from the Navier-Stokes equation, while leaving other modes unaffected, required a nontrivial conditional average. I worked on this with two of my students, and we formulated a conditional average, along with a means of approximating it, which led to a better theory: see references [2]-[4] and also see [5]

However, in recent years I was reading some lecture notes by Saffman from the sixties [6], and I saw that he had made a similar criticism of theories such as that of Heisenberg (see Section 2.8 of the book [7]), which represented the effect on lower wavenumbers of inertial transfer to higher wavenumbers by some model. In Heisenberg's case, this involved an eddy viscosity hypothesis, but Saffman made the general criticism of all models of this type, which ran as follows.

\`Conceptually, the theories are also objectionable as they ignore the phases of the Fourier components and almost regard Fourier components as having a real physical existence, rather than being a mathematical representation of the motion.'

He doesn't explain any further what he means by this, nor does he mention the lack of scale separation of the kind that justifies the calculation of the viscosity in the kinetic theory of gases. I will return to these points in the next blog post, but what really fascinates me is the cultural dissonance, which seems to take us into the realm of the philosophy of science. This too, I hope to return to in a future post.

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