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Having shown that the Edwards theory is Markovian, our present task is to show that Kraichnan's DIA and Herring's SCF are closely related to the Edwards theory.
However, we should first note that, in the case of the DIA, one can see its Markovian nature by considering its prediction for $T(k,t)$, and this was pointed out by no less a person than Kraichnan himself in 1959 [1]. We may quote the relevant passage as follows:

'The net flow is the resultant of these *absorption* and *emission* terms. It will be noticed that in contrast to the absorption term, the emission terms are proportional to $E(k)$. This
indicates that the energy exchange acts to maintain equilibrium. If the
spectrum level were suddenly raised to much higher than the equilibrium
value in a narrow neighbourhood of k, the emission terms would be
greatly increased while the absorption term would be little affected,
thus energy would be drained from the neighbourhood and equilibrium
re-established. The structure of the emission and absorption terms is
such that we may expect the energy flow to be from strongly to

weakly
excited modes, in accord with general statistical mechanical principles.'

Note that the *absorption* term is what Edwards would call the *input to mode* k *from all other modes*, while the *emission* term is the *loss from mode* k.

Kraichnan's argument here is essentially a more elaborate version of that due to Edwards, and presents what is very much a Markovian picture of turbulence energy transfer. But, in later years, numerical experiments based on high-resolution direct numerical simulations did not bear that picture out. In particular, we note the investigation by Kuczaj *et al* [2].

Going back to the relationships between theories, in 1964 Kraichnan [3] showed that if one assumed that the time-correlation and response functions were assumed to take exponential forms (with the same decay parameter $\omega(k,t)$), then the DIA reduced to the Edwards theory, although with only two ωs in the denominator of the equation for ω, rather than the three such parameters as found in the Edwards case: see equations (4) and (5) in the previous blog. Thus the arguments used to demonstrate the Markovian nature of the Edwards theory do not actually work for the single-time stationary form of DIA. See also [4], Section 6.2.6. All we establish by this procedure is that the theories are cognate: that is, they have identical equations for the energy spectrum and similar equations for the response function.

Herring's SCF has been discussed at some length in Section 6.3 of the book [4]. In time-independent form, it is identical to the DIA with assumed exponential time-dependences. The relationship between the two theories can also be demonstrated for the two-time case. The case for the SCF being classified as Markovian seems strong to me. However, there is some additional evidence from other self-consistent field theories.

Balescu and Senatorski [5] actually formulated the problem in terms of a master equation and then treated it perturbatively. Summation of certain classes of diagrams led to the recovery of Herring's SCF. For completeness, we should also mention the work of Phythian [6], whose self-consistent method resembled those of Edwards and Herring. However his introduction of a infinitesimal response function, like that of DIA, meant that his theory ended up re-deriving the DIA equations.

In the next post we will examine the question of how the Edwards theory came to be Markovian. In particular, we will answer the question: what were the relevant assumptions made by Edwards?

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