

The non-Markovian nature of turbulence 1: A puzzling aspect of the pioneering two-point closures.

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When I began my postgraduate research on turbulence in 1966, the field had just gone through a very exciting phase of new developments. But there was a snag. These exciting new theories which seemed so promising were not quite correct. They had been found to be incompatible with the Kolmogorov spectrum.

This realisation had come about in stages. When Kraichnan published his pioneering paper in 1959 [1], he carried out an approximate analysis and concluded that his new theory (the direct interaction approximation, or DIA as it is universally known) predicted an inertial range energy spectrum proportional to $k^{-3/2}$. He also concluded that the experimental results available at the time were not sufficiently accurate to distinguish between his result and the well-known Kolmogorov $k^{-5/3}$ form. However, this situation had changed by 1962, with the publication of the remarkable results of Grant et al [2], which exhibited a clear $-5/3$ power law over many decades of wavenumber range.

In 1964, Edwards published a self-consistent field theory which, unlike Kraichnan's DIA, was restricted to single-time correlations [3]. This too turned out to be incompatible with the Kolmogorov spectrum [4]. Edwards attributed the problem to an infra-red divergence in the limit of infinite Reynolds number which, although a different explanation from

Kraichnan's, at least also suggested that the problem was associated with low wavenumber behaviour. In 1965, Herring published a self-consistent field theory [5], which was comparable to that of Edwards, although the equation for the renormalized viscosity differed slightly, but not sufficiently to eliminate the infra-red divergence. In passing, I would note that Herring's self-consistent field method was more general than that of Edwards, and that is a point which I will refer to in later posts in the present series. Also, for completeness, I should mention that Herring later extended his theory to the two-time case and this was found to be closely related to the DIA of Kraichnan [6].

Kraichnan, in a series of papers, responded to this situation by developing variants of his method in Lagrangian coordinates (later on, in collaboration with Herring); and later Lagrangian methods were introduced by Kaneda, Kida & Goto, and most recently Okumura. My own approach began in 1974, in correcting the Edwards theory, which involved the introduction of the local energy transfer (LET) theory and retained the Eulerian coordinate system. All of these theories are compatible with the Kolmogorov spectrum.

My point now, is really one of taxonomy, although it is nonetheless fundamental for all that. How should we classify the theories in order to distinguish between those which are compatible with Kolmogorov and those which are not? In my 1990 book [7], I resorted to the pragmatic classification: *Theories of the first kind* and *Theories of the second kind*; along with a nod to a popular film title! Actually, in recent times, the answer to this question has become apparent, along with the realisation that it has been hiding in plain sight all this time. The clue lies in the Edwards theory and that is the aspect that we shall develop in this series of posts.

The discussion above does not do justice to everything that was going on in this field in the 1960/70s. For instance, I could have mentioned the formalism of Wyld and the well-known

EDQNM. Discussions of these, and many more, will be found in my book cited above as [7]. Also, the most recent significant research papers in this field are McComb & Yoffe [8] in 2017 and Okamura [9] in 2018.

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