Work in progress.

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In my blog of 13 August 2020 I posted a `to-do list' that dated from November 2009. None of these jobs ever got done, because other jobs cropped up which had greater priority. I'm fairly confident that this won't happen with my current `to-do list' as I see all these jobs as very important and, in a sense, as rounding off my lifetime's work in turbulence. The list follows below:

[A] Extension of my 2009 analysis of the Kolmogorov spectrum for the stationary case [1] to the case of free decay. It has become increasingly clear in recent years that there are non-trivial differences between stationary isotropic turbulence and freely decaying isotropic turbulence (and grid-generated turbulence is something else again!). As this analysis expresses the pre-factor (i.e. the Kolmogorov constant) in terms of an average over the phases of the system, it is of interest to see whether the peculiarities of free decay affect the pre-factor or the power law (or indeed both).

[**B] Turbulent inertial transfer as a non-Markovian stochastic process and the implications for statistical closures.** In 1974 [2] I diagnosed the failure of the Edwards single-time theory (and by extension Kraichnan's two-time DIA) as being due to their dividing the transfer spectrum into *input* and *output*. The basis of my local energy transfer (LET) theory was to recognise that at some wavenumbers the *entire* transfer spectrum behaved as an *input* while at other wavenumbers it behaved as an *outpu*t. Subsequently I extended the LET theory to the two-time case by heuristic methods and this formulation was developed by myself and others over many years. However in 2017 [3, 4] I extended the general self-consistent field method of Sam Edwards to the two-time case and re-derived the LET in a more formal way. However, the puzzle was this: why did the Edwards procedure give the wrong answer for the single-time case, but not for the two-time case? I realised at the time (i.e. in 2017) that Edwards had over-determined his base distribution and that his base operator was of unnecessarily high order (see [4]), but it was only recently that the penny dropped and I realised that by specifying the Fokker-Planck operator, Sam had effectively made a Markovian approximation. This needs to be written up in detail in the hope of throwing some light on the behaviour of statistical closure theories and that is my most urgent task. Please note that the letter `M' in EDQNM refers to the fact that it is Markovian in time.

[**C] Characteristic decay times of the two-time, two-point Eulerian correlation function and the implications for closures.** This is a very old topic which still receives attention: for instance, see [5, 6]. I have intended to get to grips with this for many years, as I have some concerns about the way that it is applied to statistical closures, beginning with the work of Kraichnan on DIA. One suspicion that I have is that the form of scaling is different in the stationary and freely-decaying cases; but I have not seen this point mentioned in the literature.

[**D] Reconsideration of renormalization methods in the light of the transient behaviour of the Euler equation.** I have posted five blogs with remarks on this topic, beginning on the 19 May 2022. My intention now is to combine these remarks into some more or less coherent analysis, as I believe they support my long-held suspicion (more suspicion!) that there are problems with the way in which stirring forces are used in formulating perturbation theories of the Navier-Stokes equations. Of course it is natural to study a dynamical system subject to a random force, but in the case of turbulence the force creates the system as well as sustaining it against dissipation.

This programme should keep me pretty busy so I don't expect to post blogs over the next month or two. However, by the autumn

I hope to return to at least intermittent postings.

[1] David McComb. Scale-invariance and the inertial-range spectrum in three-dimensional stationary, isotropic turbulence. J. Phys. A: Math. Theor., 42:125501, 2009.
[2] W. D. McComb. A local energy transfer theory of isotropic turbulence. J.Phys.A, 7(5):632, 1974.
[3] David McComb. A fluctuation-relaxation relation for homogeneous, isotropic turbulence. J. Phys. A: Math. Theor., 42:175501, 2009.
[4] W. D. McComb and S. R. Yoffe. A formal derivation of the local energy transfer (LET) theory of homogeneous turbulence. J. Phys. A: Math. Theor., 50:375501, 2017.
[5] G. He, G. Jin, and Y. Yang. Space-time correlations and dynamic coupling in turbulent flows. Ann. Rev. Fluid Mech., 49:51, 2017.
[6] A. Gorbunova, G. Balarac, L. Canet, G. Eyink, and V. Rossetto. Spatio-temporal correlations in three-dimensional homogeneous and isotropic turbulence. Phys. Fluids, 33:045114, 2021.