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In the preceding posts we have discussed the fact that the Euler equation can behave like the Navier-Stokes equation (NSE) as a transient for sufficiently short times [1], [2]. It has been found that spectra at lower wavenumbers are very similar to those of turbulence, and there appears to be a transfer of energy to the `thermal' modes at higher wavenumbers. This raises some rather intriguing questions about the general class of renormalized perturbation theories which are often interpreted as renormalizing the fluid viscosity. As these theories are broadly in quite good qualitative and quantitative agreement with the observed behaviour of the NSE, they should also be in good agreement with the spectrally-truncated Euler equation, which of course is inviscid. So in this case there is nothing to renormalize!

In effect this latter point has already been demonstrated, in that [1] was based on direct numerical simulation of the Euler equation and [2] used the EDQN model with the viscosity set equal to zero. So this raises doubts about the concept of a renormalized fluid viscosity as an interpretation of the two-point statistical closure theories. As indicated at the end of the previous post, it may be helpful to consider a case where the renormalization of the fluid viscosity is central to the method and therefore unambiguous. This is provided by the application of renormalization group (RG) to turbulence. A background discussion of this method may be found in [3] and a schematic outline was given in my blog post of 7 May 2020. Here we will just summarise a few points.

Consider isotropic turbulence with wavenumber modes in the range $0\leq k\leq k_{max}$. The basic idea is to average out the modes with $k_1\leq k \leq k_{max}$, while keeping those modes with $0\leq k\leq k_1$ constant. It should be emphasised that such an average is a $\emph{conditional}$ average: it is not the same as the usual ensemble or time average. Once calculated, this average can be added to the molecular viscosity in order to represent the effect of the eliminated modes by an effective viscosity on the retained modes. Then the variables are all scaled (Kolmogorov scaling) on the increased viscosity; and the process repeated for a new cut-off wavenumber $k_2< k_1$; and so on, until the effective viscosity ceases to change. The result is a scale-dependent renormalized viscosity.

Now this appears to round off my series of posts on this topic quite well. There is no viscosity in the Euler equation and so we do not have a starting point for RG. It is as simple as that. Any attempt to categorise the energy sink provided by the equilibrium modes by an effective viscosity still does not appear to provide a starting point for RG. On the other hand, unlike in the so-called renormalized perturbation theories, there is no question about the fact that the kinematic viscosity of the fluid is renormalized.

My overall conclusion is a rather vague and open-ended one. Namely, that it would be interesting to consider all the renormalization approaches to turbulence very much in the context of how they look when applied to the Euler equation as well as the NSE, and I hope to make this the subject of further work. Lastly, before finishing I should enter a caveat about RG and also correct a typographical error.

$\emph{Caveat}$: The choice of the wavenumber k_{max} is crucial. The pioneering applications of RG to random fluid motion chose it to be small enough to exclude the turbulence cascade and found a trivial fixed point as $k \rightarrow 0$. This choice rendered the conditional average trivial, as it restricted the formulation to perturbation theory using Gaussian averages, and of course Gaussian distributions factorize. Unfortunately many supposed applications to NSE turbulence also treated the conditional average as trivial. In fact one must choose k_{max} to be large enough to capture all the dissipation, at least to a good approximation.

$\emph{Correction}$: The last word of the first paragraph of my post on 19 May 2022 should have been `viscosity' not velocity. The correction has been made online.

[1] Cyril Cichowlas, Pauline Bonatti, Fabrice Debbasch, and Marc Brachet. Effective Dissipation and Turbulence in Spectrally Truncated Euler Flows. Phys. Rev. Lett., 95:264502, 2005.

[2] W. J. T. Bos and J.-P. Bertoglio. Dynamics of spectrally truncated inviscid turbulence. Phys. Fluids, 18:071701, 2006.

[3] W. D. McComb. The Physics of Fluid Turbulence. Oxford University Press, 1990.