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The term renormalization comes from particle physics but the concept originated in condensed matter physics and indeed could be said to originate in the study of turbulence in the late 19th and early 20th centuries. It has become a dominant theme in statistical theories of turbulence since the mid-20th century, and a very simple summary of this can be found in my post of 16 April 2020, which includes the sentence: `In the case of turbulence, it is probably quite widely recognized nowadays that an effective viscosity may be interpreted as a renormalization of the fluid kinematic viscosity.' Some further discussion (along with references) may be found in my posts of 30 April and 7 May 2020, but the point that concerns me here, is how can renormalization apply to the Euler equation when its relationship to the Navier-Stokes equation (NSE) corresponds to zero viscosity?

It is well known that a randomly excited and spectrally truncated Euler system corresponds to an equilibrium ensemble in statistical mechanics. This means that it must exhibit energy equipartition at long times (depending on initial conditions) with the spectral energy density $C(k) = A$, where A is constant, and therefore the energy spectrum taking the form $E(k) \sim A k^2$. Indeed this was demonstrated as long ago as 1964 by Kraichnan in the course of testing his DIA statistical closure [1]. However, in 1993, She and Jackson studied a constrained Euler system in the context of reducing the number of degrees of freedom needed to describe Navier-Stokes turbulence [2]. This involves an Euler equation restricted to wavenumber modes $k_{min}\leq k \leq k_{max}$ which is embedded in a forced NSE system, with nonlinear transfer of energy in from the forced modes with $k<k_{min}$

and nonlinear dissipation of energy to modes with $k>k_{max}$, where viscous dissipation is present. This is a very interesting paper which I mention here for completeness and hope to return to at some later time. For the moment I want to concentrate on two rather simpler, but still important, studies of the incompressible Euler equation [3], [4].

In physical terms, it is well known that the presence of the viscous term in the NSE, with its k^2 weighting, breaks the symmetry of the nonlinear term common to both equations, and ensures a mean flux of energy in the direction of increasing wavenumber. This symmetry can also be broken by adopting as initial condition some energy spectrum which does not correspond to the equipartition solution. The resulting evolution of a system peaked initially near, but not at, the origin is shown in [1], along with a good discussion of the behaviour of the Euler equation as related to the NSE. Evidently the Euler equation may behave like the NSE as a transient, ultimately tending to equipartition. This behaviour has been studied by Cichowlas et al [3], using direct numerical simulation; and by Bos and Bertoglio [4], using the EDQNM spectral closure. They both find long-lived transients in which at smaller wavenumbers there is a Kolmogorov-type $k^{-5/3}$ spectrum and at higher wavenumbers an equipartition k^2 spectrum. In both cases, the equipartition range is seen as acting as a sink, and hence giving rise to an effect like molecular viscosity.

In [4], as in [2], there is some consideration of the relevance to large-eddy simulation, but it should be noted that in both investigations the explicit scales are not subject to molecular viscosity or its analogue. For sake of contrast we note that the operational treatment of NSE turbulence by Young and McComb [5] provides a subgrid sink for explicit modes which are governed by the NSE. It may not be a huge difference in practice, but it is important to be precise about these matters.

However, in the present context the really interesting aspect of [3] and [4] is that, in the absence of viscosity they obtain the sort of turbulent spectrum which may be interpreted in terms of an effective turbulent viscosity, and hence in terms of self-renormalization. In the next post, we will examine this further, beginning with a more detailed look at what is meant by the term renormalization.

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