The question of notation.

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In recent years, when I specify the velocity field for turbulence, I invariably add a word of explanation about my use of Greek letters for Cartesian tensor indices. I point out that these Greek indices should not be confused with those used in four-space for four-dimensional tensors, as encountered in Einstein’s Relativity. I think that I began do this round about the time I retired in 2006 and at the same time began looking at problems in phenomenology. Previously I had just followed Sam Edwards, who had been my PhD supervisor, because it seemed such a very good idea. By reserving Greek letters for indices, one could use letters like k, j, l, p $\dots$ for wavenumbers, which reduced the number of primed or multiply-primed variables needed in perturbation theory.

Presumably it had occurred to me that a different audience might not be familiar with this convention, or perhaps some referee rejected a paper because he didn’t know what Greek letters were [1]? In any case, it was only recently that it occurred to me that Kolmogorov actually uses this convention too. In fact in the paper that I refer to as Kolmogorov 41A [2], one finds the first sentence: ‘We shall denote by $$u_{\alpha} (P) = u_{\alpha} (x_1,x_2,x_3), \quad \alpha=1,\,2,\,3,$$ the components of the velocity at the moment t at the point with rectangular Cartesian coordinates x_1,x_2,x_3. So in future, I could say ‘as used by Kolmogorov’.

Kolmogorov also introduced the second-order and third-order longitudinal structure functions as $B_{dd}(r,t)$ and $B_{ddd}(r,t)$ (the latter appearing in K41B [3]), and others followed similar schemes, with the number of subscript ds increasing with order. This was potentially clumsy, and when experimentalists became able to measure high-order moments in the 1970s, they resorted to the notation $S_n(r,t)$. That is, S for ‘structure function’ and integer n for order, which is nicely compact.

During the sixties, statistical turbulence theories used a variety of notations. Unfortunately, for some people a quest for an original approach to a well known problem can begin with a new notation. On one occasion, I remember thinking that I didn’t even know how to pronounce the strange symbol that one optimistic theorist had used for the vertex operator of the Navier-Stokes equation. That was back in the early 1970s and it is still somewhere in my office filing cabinets. I don’t think I missed anything significant by not reading it!

Notational changes should be undertaken with caution. During the late 1990s I was just about the only person working on statistical closure theory (at least, in Eulerian coordinates) and I decided to adopt an emerging convention in dynamical systems theory. That is, I decided to represent all correlations by C and response tensors by R.

The only other change I made was to change the symbol for the transverse projection operator to Kraichnan’s use of P, from Edwards’s use of D. The result is, in my view, a notationally more elegant formalism; and perhaps if people again start taking an interest in renormalized perturbation theories and renormalization group, this would get them off to a good start.

However, there can be more to a formalism than just the notation. The true distinction between the two really lies in the *formulation*. Starting with the basic vector triad $\mathbf{k},\mathbf{j},\mathbf{l}$, Edwards used the triangle condition to eliminate the third vector as $\mathbf{l}=\mathbf{k}-\mathbf{j}$. This was done by others, but in the context of the statistical theories virtually everyone followed Kraichnan’s much more complicated approach, in which he retained the three scalar magnitudes and imposed on all sums/integrals the constraint that they should always add up to a triangle. The resulting formulation is more opaque, more difficult to compute and does not permit symmetries to be deduced by simple inspection. Yet for some reason virtually everyone follows it, particularly obviously in the use of EDQNM as a model for applications. A concise account of the two different formalisms can be found in Section 3.5 of the book [4].

[1] Just joking! I’ve never had a paper rejected for that reason, but some rejections over the years have not been a great deal more sensible.

[2] A. N. Kolmogorov. The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. C. R. Acad. Sci. URSS, 30:301, 1941.

[3] A. N. Kolmogorov. Dissipation of energy in locally isotropic turbulence. C. R. Acad. Sci. URSS, 32:16, 1941.

[4] W. David McComb. Homogeneous, Isotropic Turbulence: Phenomenology, Renormalization and Statistical Closures. Oxford University Press, 2014.