Turbulence in a box.

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When the turbulence theories of Kraichnan, Edwards, Herring, and so on, began attracting attention in the 1960s, they also attracted attention to the underlying ideas of homogeneity, isotropy, and Fourier analysis of the equations of motion. These must have seemed very exotic notions to the fluid dynamicists and engineers who worked on single-point models of the closure problem posed by the Reynolds equation. Particularly, when the theoretical physicists putting forward these new theories had a tendency to write in the language of the relatively new topic of quantum field theory or possibly the even newer statistical field theory. In fact, the only aspect of this new approach that some people working in the field were apparently able to grasp was the fact that the turbulence was in a box, rather than in a pipe or wake or shear layer.

I became aware of this situation when submitting papers in the early 1970s, when I encountered referees who would begin their report with: 'the author invokes *the turbulence in a box concept*'. This seemed to me to have ominous overtones. I mean, why comment on it? No one working in the field did: it was taken as quite natural by the theorists. However, in due course it invariably turned out that the referee didn't think that my paper should be published. Reason? Apparently just the unfamiliarity of the approach. Later on, with the subject of turbulence theory having reached an impasse, they clearly felt quite confident in turning it down. I have written before on my experiences of this kind of refereeing (see, for example, my post of 20 Feb 2020).

Another example of turbulence in a box is the direct numerical simulation of isotropic turbulence, where the Navier-Stokes equations are discretised in a cubical box in terms of a discrete Fourier transform of the velocity field. Since Orszag

and Patterson's pioneering development of the pseudo-spectral method [1] in 1972, the simulation of isotropic turbulence has grown in parallel with the growth of computers; and, in the last few decades, it has become quite an everyday activity in turbulence research. So, now we might expect *box turbulence* to take its place alongside pipe turbulence, jet turbulence and so on, in the jargon of the subject?

In fact this doesn't seem to have happened. However, less than twenty years ago, a paper appeared which referred to simulation in *a periodic box* [2], and since then I have seen references to this in microscopic physics, where the simulations are of molecular systems. I'm not sure why the nature of the box is worth mentioning. It is, after all, a commonplace fact of Fourier analysis, that representation of a non-periodic function in a finite interval requires an assumption of periodic behaviour outside the interval. Much stranger than this is that I am now seeing references to *periodic turbulence* as, apparently, denoting isotropic turbulence that has been simulated in a periodic box. This does not seem helpful! To most people in the field, periodic turbulence means turbulence that is modulated periodically in time or space. That is, the sort of turbulence that might be found in rotating machinery or perhaps a coherent structure [3]. We have to hope that this usage does not catch on.

[1] S. A. Orszag and G. S. Patterson. Numerical simulation of three-dimensional homogeneous isotropic turbulence. Phys.Rev.Lett, 28:76, 1972.
[2] Y. Kaneda, T. Ishihara, M. Yokokawa, K. Itakura, and A. Uno. Energy dissipation and energy spectrum in high resolution direct numerical simulations of turbulence in a periodic box. Phys. Fluids, 15:L21, 2003.
[3] W. D. McComb. The Physics of Fluid Turbulence. Oxford University Press, 1990.