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In last week's post I reiterated the argument that the existence of isotropy implies homogeneity. However, Alex Liberzon commented that there could be inhomogeneous flows that exhibited isotropy on scales that were small compared to the overall size of the flow. This comment has the great merit of drawing attention to the difference between a purely theoretical formulation and one dealing with a real practical situation. In my reply, I mentioned that Kolmogorov had introduced the concept of l*ocal isotropy*, which supported the view that Alex had put forward. So I thought it would be interesting to look in detail again at what Kolmogorov had actually said. Incidentally, Kolmogorov said it in 1941 but for the convenience of readers I have given the later references, as reprinted in the Proceedings of the Royal Society.

Now, although I like to restrict the problem to purely isotropic turbulence, where it still remains controversial in that many people believe in intermittency corrections or anomalous exponents, Kolmogorov actually put forward a theory of turbulence in general. He argued that a cascade as envisaged by Richardson could lead to a range of scales where the turbulence becomes *locally homogeneous*. In [1], which I refer to as K41A, he put forward two definitions, which I shall paraphrase rather than quote exactly.

The first of these is as follows: `**Definition 1**. The turbulence is called *locally homogeneous* in the domain G if the probability distribution of the velocity differences is independent of the origin of coordinates in space, time and velocity, providing that all such points are contained within the domain G.'

We should note that this includes homogeneity in time as well as in space. In other words, Kolmogorov was assuming *local stationarity* as well.

Then his second definition is: `**Definition 2**. The turbulence is called locally isotropic in the domain G, *if it is homogeneous* and if, besides, the distribution laws mentioned in Definition 1 are invariant with respect to rotations and reflections of the original system of coordinate axes $(x_1,\,x_2\,x_3)$.'

Note that the emphasis is mine.

Kolmogorov then compared his definition of isotropy to that of Taylor, as introduced in 1935. He stated that his definition is narrower, because he also requires local stationarity, but wider in that it applies to the distribution of the velocity differences, and not to the velocities themselves. Later on, when he derived the so-called '$4/5$' law [2], he had already made the assumption that the time-derivative term could be neglected, and simply quoted the Karman-Howarth equation without it: see equation (3) in [2].

The question then arises, how far do these assumptions apply in any real flow? In my post of 11th February 2021, I conjectured that this might be a matter of the macroscopic symmetry of the flow. For instance, the Kolmogorov picture might apply better in plane channel flow that in plane Couette flow. I plan to return to this point some time.

[1] A. N. Kolmogorov. The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. Proc. Roy Soc. Lond., 434:9-13, 1991.
[2] A. N. Kolmogorov. Dissipation of energy in locally isotropic turbulence. Proc. Roy Soc. Lond., 434:15-17, 1991.