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The current interest in Onsager's conjecture (see my blog of 23 September 2021) has sparked my interest in the nature of turbulent dissipation. Essentially a fluid only moves because a force acts on it and does work to maintain it in motion. The effect of viscosity is to convert this kinetic energy of macroscopic motion into random molecular motion, which is perceived as heat. If there is turbulence, this acts to transfer the macroscopic kinetic energy to progressively smaller scales, where the steeper velocity gradients can dissipate it as heat.

This all seems quite straightforward and well understood. However, Onsager's conjecture, as a matter of physics, is less easily understood. It interprets the infinite Reynolds number limit as being when the continuum nature of the fluid breaks down. It also implies that, when the Reynolds number becomes very large, the Navier-Stokes equation somehow becomes the Euler equation; which, despite its inviscid nature, satisfactorily accounts for the dissipation. It can do this (supposedly) because it has lost its property of conserving energy. In turn, this is supposed to happen because the velocity is no longer a continuous and differentiable field. Of course there does not seem to be any mechanism for turning the dissipated energy into heat, so the thermodynamic aspects of this process look distinctly dodgy.

There are two other cases where macroscopic kinetic energy is not turned into heat.

The first of these is in large-eddy simulation, which has for many years been widely studied for its practical significance.

This of course is not a physical situation. It is purely a method of simulating turbulence numerically without being able to resolve all the scales: an introduction can be found in [1]. The central problem is to model the flow of energy to the scales which are too small to be resolved: the so-called *subgrid drain*. Various models have been studied for the subgrid viscosity, while a novel approach is the operational method of Young and McComb [2]. In this latter, an algorithm is used to feed back energy into the resolved modes, such that the spectral shape is kept constant. In fact this method can be interpreted in terms of an effective subgrid viscosity which is very similar to that found in conventional simulations when a large-eddy simulation is compared to a fully resolved one. But, so far as I know, no one has considered modelling the temperature rise that would be due to the viscous dissipation in these cases.

The second case is the direct simulation of the Euler equation. Such simulations can only lead to thermal equilibrium but naturally the simulations must be truncated to a finite number of modes, to avoid having an infinite amount of energy. However, in 2005, some interesting transient behaviour was been found in truncated Euler simulations [3] and confirmed the following year by the use of a closure approximation [4]. These simulations may be divided in terms of their energy spectra into two spectral ranges: a Kolmogorov range and an equipartition range. A buffer range in between these two is described by Bos and Bertoglio as a '*quasi-dissipative*' zone, which is another example of non-viscous dissipation. However, it can only exist for a finite time and ultimately the system must move to thermal equilibrium.

I think it would be interesting to see one of the proponents of Onsager's conjecture explain the simple physics of how the conjectured situation came about with increasing Reynolds number. All the mathematical expressions you need to do that are available. But I don't think I will see that any time soon!

[1] W. D. McComb. The Physics of Fluid Turbulence. Oxford University Press, 1990.
[2] A. J. Young and W. D. McComb. Effective viscosity due to local turbulence interactions near the cutoff wavenumber in a constrained numerical simulation. J. Phys. A, 33:133-139, 2000.
[3] Cyril Cichowlas, Pauline Bonatti, Fabrice Debbasch, and Marc Brachet. Effective Dissipation and Turbulence in Spectrally Truncated Euler Flows. Phys. Rev. Lett., 95:264502, 2005.
[4] W. J. T. Bos and J.-P. Bertoglio. Dynamics of spectrally truncated inviscid turbulence. Phys. Fluids, 18:071701, 2006.