Why am I so concerned about Onsager’s so-called conjecture?

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Staycation post No 3. I will be out of the virtual office until 30 August.

In recent years, Onsager’s (1949) paper on turbulence has been rediscovered and its eccentricities promoted enthusiastically, despite the fact that they are at odds with much well-established research in turbulence, beginning with Batchelor, Kraichnan, Edwards, and so on. In particular, a bizarre notion has taken hold that the Euler equation corresponds to the zero-viscosity limit of the Navier-Stokes equations and can be made dissipative, in defiance of the basic physics, by some mysterious alteration of the mathematics. The previous two posts refer to this.
I have been intending to write about this for some time, but the present paper [1] was prompted by an email that I received late in 2019 from MSRI, Berkeley. This was an advance announcement of a Program: ‘*Mathematical problems in fluid dynamics*’, to take place in the first half of 2021. I quote from the description as follows:

‘The fundamental equations in this area are the well-known Euler equations for inviscid fluids and the Navier-Stokes equations for the (sic) viscous fluids. Relating the two is the problem of the zero-viscosity limit and its connection to the phenomena of turbulence.’

The second sentence is nonsense and runs counter to all the conventions of fluid dynamics, where it has long been known that the relationship between the two equations is obtained by setting the viscosity equal to zero. The infinite Reynolds number limit, in contrast, is observed as an asymptotic behaviour of the Navier-Stokes equation; which, even at high Reynolds numbers, remains the Navier-Stokes equation.

I was appalled by the thought of young mathematicians being taught such unrepresentative and incorrect material. This is what provided my immediate motivation for writing the present paper. The first version of this paper was put on the arXiv on 12 December 2020.

In January of this year, I received from MSRI the final notification of this program. The wording had changed, and after some unexceptional statements about the equations of motion it read:

‘Open problems and connections to related branches of mathematics will be discussed, including the phenomena of turbulence and the zero-viscosity limit. Both theoretical and numerical aspects of these topics will be considered.’

Perhaps it is just a coincidence that this change should follow the arXiv publication of [1], but at least their statement about their course is no longer manifestly false; although much still depends on what was actually taught. It may be noted that Figure 2 of [1] (also see the previous post) shows the onset of scale invariance and, in effect, the zero-viscosity limit, in a direct numerical simulation at a Taylor-Reynolds number of about one hundred. This is the *physical* infinite Reynolds number limit as it occurs in real fluids.

Another aspect of the influence of Onsager is the use of the term *dissipation anomaly* which is used instead of what some call the *dissipation law*. If one criticises the term, the mathematicians seem to believe that one is denying the existence of the effect. Not so. At Edinburgh we have worked on the establishing the existence of the dissipation law and also have elucidated it as arising from the Richardson-Kolmogorov picture [2], [3]. It is a real physical effect and there is nothing anomalous about it.

[1] W. D. McComb and S. R. Yoffe. The infinite Reynolds number limit and the quasi-dissipative anomaly. arXiv:2012.05614v2[physics.flu-dyn], 2021.
[2] W. David McComb, Arjun Berera, Matthew Salewski, and Sam R. Yoffe.
Taylor’s (1935) dissipation surrogate reinterpreted. Phys. Fluids, 22:61704,
2010.

[3] W. D. McComb, A. Berera, S. R. Yoffe, and M. F. Linkmann. Energy transfer and dissipation in forced isotropic turbulence. Phys. Rev. E, 91:043013, 2015.