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In 1971, when I began as a lecturer in Engineering Science at Edinburgh, my degree in physics provided me with no basis for teaching fluid dynamics. I had met the concept of the convective derivative in statistical mechanics, as part of the derivation of the Liouville equation, and that was about it. And of course the turbulence theory of my PhD was part of what we now call statistical field theory. Towards the end of autumn term, I was due to take over the final-year fluids course, but fortunately a research student who worked as a lab demonstrator for me had previously taken the course and kindly lent me his copy of the lecture notes. However, in my first year, I was never more than one lecture ahead of the students!

This grounding in the subject was reinforced by practical experience, when I began doing experimental work on drag reduction by additives and on particle diffusion. It also allowed me to recover quickly from an initial puzzlement, when I saw a paper in JFM which proposed that the occurrence of streamwise vorticity could be taken as a signal of turbulence in duct flow.

Later on, I learned that this idea could be extended to give a plausible picture of the turbulent bursting process, and a discussion can be found in Section 11.4.3 of my book [1], where the development of Λ vortices is illustrated in Fig. 11.1. In the book, this is preceded by a treatment of the boundary layer on a flat plate in Section 1.4, which can help us to understand the basic idea as follows. Suppose we have a fluid moving with constant velocity U_1, incident on a flat plate lying in the (x_1,x_3) plane with its leading edge at $x_1=0$. Vorticity is generated at this point due to the noslip boundary condition, and diffuses out normal to the plate in the x_2 direction, resulting in a velocity field of the form $u_1(x_2)$, in the boundary layer. We can visualize the sense of the vorticity vector by imagining the effect of a small portion of the fluid becoming solidified. That part nearest the plate will slow down, the ‘solid body’ will rotate, and the spin vector will point in the x_3 direction. This is the only component of vorticity in the system.

The occurrence of vorticity in the other two directions must be a consequence of instability and almost certainly begins with vorticity building up in the x_1 direction due to edge effects. That is, in practice, the plate must be of finite extent in the cross-stream or x_3 direction. A turbulence transition could not occur if the plate (as normally assumed for pedagogic purposes) were of infinite extent. This provides an unequivocal criterion for the occurrence of the *transition* to turbulence, but there is still the question of when the turbulence is in some sense well-developed. And of course other flows may require other criteria.

The question of whether a flow is turbulent or not became something of an issue in the 1980s/90s, when there was a growing interest in applying Renormalization Group (RG) to turbulence. The pioneering work on applying RG to randomly stirred fluid motion was reported by Forster, Nelson and Stephen [2] in 1976, and you should note from the title of their first paper that the word ‘turbulence’ does not appear. Their work was restricted to showing that there was a fixed point of the RG transformations in the limit of zero wavenumbers (i.e. ‘large wavelengths’).

The main drive in turbulence research is always towards applications, and inevitably pressure developed to seek ways of extending the work of Forster *et al.* to turbulence. In the process a distinction grew up between *‘stirred fluid motion’* and so-called *‘Navier-Stokes turbulence’*. The latter should be described by the spectral energy balance known as the Lin equation, whereas the former just reflects its Gaussian forcing. Nowadays, in physics, the distinction has settled down to *‘stirred hydrodynamics’* and just plain turbulence!

The difficulty of defining turbulence in a concise way remains, but some light can be shed on these earlier controversies by considering a more recent discovery that we made at Edinburgh. This was the result that a dynamical system consisting of the Navier-Stokes equations forced by the combination of an initial Gaussian field and a negative damping term, will at very low Reynolds numbers become non-turbulent and take the form of a Beltrami flow [3]. In this paper, we emphasised that at early times the transfer spectrum $T(k,t)$ has the behaviour typically found in simulations of isotropic turbulence but at later times tends to zero. At the same time, the energy spectrum $E(k,t)$ tends to a unimodal spectrum at $k=1$. An interesting point is that the fixed point of Forster *et al.* $k \rightarrow 0$ is cut off by our lattice, so that we observe a Beltrami flow instead.

[1] W. D. McComb. The Physics of Fluid Turbulence. Oxford University Press, 1990.
[2] D. Forster, D. R. Nelson, and M. J. Stephen. Long-time tails and the large-eddy behaviour of a randomly stirred fluid. Phys. Rev. Lett., 36(15):867-869, 1976.
[3] W. D. McComb, M. F. Linkmann, A. Berera, S. R. Yoffe, and B. Jankauskas. Self-organization and transition to turbulence in isotropic fluid motion driven by negative damping at low wavenumbers. J. Phys. A Math. Theor., 48:25FT01, 2015.