Are Kraichnan's papers difficult to read? Part 2: The DIA.

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In 2008, or thereabouts, I took part in a small conference at the Isaac Newton Institute and gave a talk on the LET theory, its relationship to DIA, and how both theories could be understood in terms of their relationship to Quasi-normality. During my talk, I was interrupted by someone in the audience, who said that I was wrong in discussing DIA as if Kraichnan's perturbation theory was the same as that of Wyld. I disagreed, and we had a short exchange of the kind '*Yes you did! No, I didn't!*', and the matter was left unresolved.

Sometime afterwards, I refreshed my memory of these matters and realised that I was wrong. Kraichnan's seminal paper [1] is not easy to understand, but he was claiming to be introducing a new type of perturbation theory, and that undoubtedly differed from Wyld's subsequent field-theoretic approach [2]. In his book on the subject, Leslie had simply chickened out and used the Wyld analysis [3]. Many of us had then followed in his tracks, but over the years (decades!) I had simply forgotten that fact. It was salutary to be reminded of it, and I duly said something about it in my later book on turbulence [4].

Again this draws attention to the danger of relying uncritically on secondary sources, but an interesting point emerged. Kraichnan made what was essentially a mean-field approximation in his theory. The fact that Wyld could show that the DIA gave identical results to the same order of truncation of conventional perturbation theory tells us that the mean-field approximation for the response function was justified; because the method of renormalization was the same for both approaches. This is of further interest, in that the recent formal derivation of the local energy-transfer (LET) theory also relies on a mean-field approximation involving the response function [5], although this is defined in a completely different way from that in DIA.

Among the select few who actually have got to grips with the new perturbation theory in [1], are my student Matthew Salewski, who did that as a preliminary to the resolution of the apparent differences between formalisms [6]; and S. Kida who revisited DIA in order to derive a Lagrangian theory e.g. see reference [7].

As regards the question which heads this post, we can leave the last word with the man himself. Kraichnan told me that on one occasion a referee had complained to him: ‘*Why are your papers so difficult to read?’* and he had replied: ‘*If you think they are hard to read, have you considered how difficult they must be to write?’.*

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[3] D. C. Leslie. Developments in the theory of turbulence. Clarendon Press, Oxford, 1973.
[4] W. David McComb. Homogeneous, Isotropic Turbulence: Phenomenology, Renormalization and Statistical Closures. Oxford University Press, 2014.
[5] W. D. McComb and S. R. Yoffe. A formal derivation of the local energy transfer (LET) theory of homogeneous turbulence. J. Phys. A: Math. Theor., 50:375501, 2017.
[6] A. Berera, M. Salewski, and W. D. McComb. Eulerian Field-Theoretic Closure Formalisms for Fluid Turbulence. Phys. Rev. E, 87:013007-1-25, 2013.
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