Does the failure to use spectral methods harm one's understanding of turbulence?

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Vacation post No. 3: I will be out of the virtual office until Monday 19 April.

As described in the previous post, traditional methods of visualising turbulence involve vaguely specified and ill-defined eddying motions whereas Fourier methods lead to a well-defined problem in many-body physics. This seems to be a perfectly straightforward situation; and one might wonder: in what way do fluid dynamicists feel that the Fourier wavenumber space representation is obscuring the physics? Given that they regard a vortex-based picture, however imprecise, as `the physics', I suspect (a suspicion based on many discussions over the years!) that the problem arises when they try to reconcile the two formulations. Of course, in an intuitive way, one may associate large wavenumbers with small spatial separations. That is, `high k' corresponds to `small r' and *vice versa*. But those attempts, which one sees from time to time, to interpret the k-space picture in terms of arbitrarily prescribed vortex motions in real space, seem positively designed to cause confusion. It is important to bear in mind that the Fourier representation *reformulates* the problem, and you should study it on its own terms, even if you long for vortices!

Does this matter? I think it does. For example, I can point to the strange situation in which (it seems) most fluid dynamicists believe that there should be intermittency corrections to the exponent of Kolmogorov's $k^{-5/3}$ energy spectrum, whereas it seems that most theoretical physicists (who work in wavenumber space) do not. The hidden point here, is that Kolmogorov worked in real space, and derived the $r^{2/3}$ form of the second-order structure function, for an intermediate range of values of r where the form of the input term and the viscous dissipation could both be neglected, thus introducing the inertial range. His theory was inconsistent; in that he then considered the structure function to depend on the dissipation rate, even although this had been excluded from the inertial range. It is this step which gives some credibility to the possibility of intermittency effects, particularly as there may be some doubt about whether or not the dissipation rate in the theory is the average value or not.

The surprising thing is that, at much the same time, Obukhov worked in k-space, and identified the conservative, inertial flux of energy through the modes as being the key quantity determining the energy spectrum in the inertial range. It follows that, with the production and dissipation being negligible in this range of wavenumbers, the flux must be constant (i.e. independent of wavenumber) in the inertial range This was later recognized by Osager. Later still, this property became widely known and for many years has been referred to by theoretical physicists as *scale invariance*. Scale invariance is a general mathematical property and can refer to various things in turbulence research. It simply means that something which might depend on an independent variable, in either real space or wavenumber space, is in fact constant. It should be emphasised that the inertial flux is an average quantity, as indeed is the energy spectrum, and any intermittency must necessarily be averaged out. In fact a modern analysis leading to the $k^{-5/3}$ spectrum would start from the Lin equation. Therefore it is hard to see how internal intermittency, which is incidentally present on *all* scales can affect this derivation.