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In the late 1970s there was an upsurge in interest in the turbulence problem among theoretical physicists. This arose out of the application of renormalization group (RG) methods to the problem of stirred fluid motion. As this problem was restricted to a very low wavenumber cutoff, these approaches had nothing to say about real fluid turbulence. Nevertheless, the work on RG stimulated a lot of speculative discussion, and one paper referred to `the mysterious stirring forces'. I found this rather unsettling, because I had been familiar with the concept of stirring forces from the start of my PhD project in 1966. Why, I wondered, did some people find them mysterious?

As time passed, I came to the conclusion that it was just lack of familiarity on the part of these theorists, although they seemed quite happy to launch into speculation on a subject that they knew very little about. (Well, it was just a conference paper!) So I was left with the feeling that one day it might be worth writing something to debunk this comment. Recently it occurred to me that it would make a good topic for a blog.

The standard form used nowadays for the stirring forces was introduced by Sam Edwards in 1964 and has its roots in the study of Brownian motion, and similar problems involving fluctuations about equilibrium. Let us consider the motion of a colloidal particle under the influence of molecular impacts in a liquid. For simplicity, we specialise to one-dimensional motion with velocity u. The particle will experience Stokes drag with coefficient η, per unit mass. Accordingly, we can use Newton's second law to write its macroscopic equation of motion as: \begin{equation} \partial u/\partial t =-\eta \,

u. \end{equation} At the microscopic level, the particle will experience the individual molecular impacts as a random force $f(t)$, say. So the microscopic equation of motion becomes: \begin{equation}\partial u/\partial t =-\eta \, u + f(t). \end{equation} This equation is known as the Langevin equation. In order to solve it, we need to specify f in terms of a physically plausible model.

We begin by noting that the average effect of the molecular impacts on the colloidal particle must be zero, thus we have: \begin{equation}\langle f(t) \rangle =0. \end{equation} As a result, the average of equation (2) reduces to equation (1), which is consistent. Then in order to represent the irregular nature of the molecular impacts, we assume that $f(t)$ is only correlated with itself at very short times $t\leq t_c$, where t_c is the duration of a collision. We can express this in terms of the autocorrelation function w as: \begin{equation} \langle f(t)f(t’) \rangle = w(t-t’), \end{equation} and \begin{equation} W(t) = \int_0^t\,w(\tau)\,d\tau, \end{equation} where \begin{equation} W(\tau)\rightarrow W = \mbox{constant}.\end{equation}

We can go on to solve the Langevin equation (2) for the short-time and long-time behaviour of the particle velocity $u(t)$, much as in Taylor’s Lagrangian analysis of turbulent diffusion. We can also derive the fluctuation-dissipation relation: see reference [1] for details.

In his self-consistent field theory of turbulence, Edwards drew various analogies with the theory of Brownian motion [2]. In particular, he went further than in equations (4) to (6), and chose the stirring forces to be instantaneously correlated with themselves; or: \begin{equation}w(t-t’) = W \delta(t-t’), \end{equation} where δ is the Dirac delta function. In the study of stochastic dynamical systems, this is known as `white noise forcing’. It allows one to express the rate at which the stirring force does work on the turbulent fluid in terms of the autocorrelation of the stirring forces [3].

It also provides a criterion for the detection of `fake theories'. These are theories which are put out by people with skill in quantum field theory and which purport to be theories of turbulence. Such theories do not engage with the established body of work in the theory of turbulence, nor do they mention how they overcome the problems that have proved to be a stumbling block for legitimate theories. Invariably, they attribute the purpose of the delta function to be to maintain Galiean invariance and clearly do not know what it is actually used for. In fact, the Navier-Stokes equations are trivially Galilean-invariant and adding an external force to them cannot destroy that [4].

[1] W. David McComb. Study Notes for Statistical Physics: A concise, unified overview of the subject. Bookboon, 2014.
[2] S. F. Edwards. The statistical dynamics of homogeneous turbulence. J. Fluid Mech., 18:239, 1964.
[3] W. D. McComb. The Physics of Fluid Turbulence. Oxford University Press, 1990.
[4] W. D. McComb. Galilean invariance and vertex renormalization. Phys. Rev. E, 71:37301, 2005.