Analogies between critical phenomena and turbulence: 1

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In the late 1970s, application of Renormalization Group (RG) to stirred fluid motion led to an upwelling of interest among theoretical physicists in the possibility of solving the notorious turbulence problem. I remember reading a conference paper which included some discussion that was rather naïve in tone. For instance, why did turbulence theorists study the energy spectrum rather than something else? Also, rather unsettlingly, there was a reference to the 'mysterious stirring forces' (*sic)*: I shall return to that comment in a future post. However, although no turbulence theory emerged from this activity, a way of thinking did, and this found a receptive audience in those members of the turbulence community who believe in intermittency corrections. In my view, one set of views is as unjustified as the other, and I shall now explain why I think this.

To understand how these views came about, we need to consider the background in critical phenomena. During the 1960s, theorists in this area began to use concepts like scaling and self-similarity to derive exact relationships between critical exponents. (In passing, I note that in fluid dynamics these tools had already been in active use for more than half a century!) In this way, the six critical exponents of a typical system could be reduced to just two to be determined. At first the gap was bridged by mean-field theory, but then RG came along and the problem was solved.

It is important to know that RG can be viewed, in some respects, as a correction to mean-field theory. As a result, theorists in this field essentially ended up taking the view: 'mean-field theory, bad! RG good!', and this had a tendency to spill over into other areas as a sort of judgement. In general this was the attitude during the 1980s/90s, and few paused to reflect that other phenomena might belong to a different universality class. For instance, should the self-consistent field theory of multi-electron atoms be ruled out, because RG is better than mean-field theory at describing the para-ferromagnetic phase transition? Fortunately, this sort of thinking has presumably died out by now, but it has left an unhelpful residue in turbulence theory.

One form of this is the assertion that the Kolmogorov ‘$-5/3$’ energy spectrum is a mean-field theory, and that an RG calculation would lead to an exponent of the form ‘$-5/3+\mu$’; precisely what the ‘intermittency correction’ enthusiasts had been saying all along! The snag with this is that the derivation of the Kolmogorov spectrum does not rely on a mean-field step, nor indeed on the invariable accompaniment of a self-consistent field step. In fact, this can be a problem in critical phenomena. People tend to refer loosely to mean-field theories, without mentioning that they are also self-consistent theories. Actually in turbulence we have various self-consistent field theories which do not predict the Kolmogorov exponent and one which does [1].

In my next post, I will develop this topic further. In the meantime, a general background account of these matters may be found in the book cited below as [2].

[1] W. D. McComb and S. R. Yoffe. A formal derivation of the local energy transfer (LET) theory of homogeneous turbulence. J. Phys. A: Math. Theor., 50:375501, 2017.

[2] W. D. McComb. Renormalization Methods: A Guide for Beginners. Oxford University Press, 2004.