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The universality of the small scales, which is predicted by the Richardson-Kolmogorov picture, is not always observed in practice; and in the previous post I conjectured that departures from this might be accounted for by differences in the spatial symmetry of the large scale flow. To take this idea a step further, I now wonder whether it would be worth exploring how the idea of universality classes could be applied to the turbulent cascade? First, I should explain what universality classes actually are.

In the study of critical phenomena, we are concerned with changes of phase or state which can occur at a critical temperature, which is invariably denoted by T_c. For instance, the transition from liquid to gas, or the transition from para- to ferromagnetism. In general, it is found that the thermodynamic variables (e.g. heat capacity, magnetic susceptibility) of a system either tend to zero, or tend to infinity, as the system approaches the critical temperature. If we represent any such macroscopic variable by $F(T)$ and introduce the reduced temperature Θ_c by \[\Theta_c = \frac{T-T_c}{T_c}.\] Then, as $T\rightarrow T_c$ and $\Theta_c \rightarrow 0$, we have \[F(\Theta_c) = A \Theta^{-n},\] where A is a constant and n is the critical exponent. Obviously the critical exponent will be negative when $F(0)=0$ and positive when $F(0)=\infty$.

Here the constant A and the critical temperature T_c depend on the details of the system at the molecular level and therefore vary from one system to another. These quantities must be determined experimentally. However, in practice it is found that sometimes different systems have the same values of critical exponents and this depends only on symmetry properties of the microscopic energy function (or Hamiltonian). When this is found to be the case, the two systems are said to be in the same *universality class*.

Accordingly, in my view it would be worth reviewing the different investigations in order to find out if one could organise results for the inertial-range exponent into some kind of universality classes, although allowance should be made for experimental error, which tends to be much greater in fluid dynamics than in microscopic physics. I would be tempted to take a look through my files, but unfortunately I remain cut off from my university office by the pandemic.

Further details about critical phenomena may be found in reference [1] below.

[1] W. D. McComb. Renormalization Methods: A Guide for Beginners. Oxford University Press, 2004.