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The concepts of *macroscopic* and *microscopic* are often borrowed, in an unacknowledged way, from physics, in order to think about the fundamentals of turbulence. By that, I mean that there is usually no explicit acknowledgement, nor indeed apparent realization, that the ratio of large scales to small scales is many orders of magnitude smaller in turbulence (which is at all scales actually a macroscopic phenomenon) than it is in microscopic physics.

This idea began with Kolmogorov in 1941, when he employed Richardson's concept of a cascade of energy from large eddies to small; to argue that, after a sufficiently large number of steps, there could be a range of eddy sizes which were statistically independent of their large-scale progenitors. In passing, it should be noted that the concept of 'eddy' can be left rather intuitive, and we could talk equally vaguely about 'scales'. However, combining the cascade idea with Taylor's earlier introduction of Fourier modes as the degrees of freedom of a turbulent system, leads to a much more satisfactory analogy with statistical physics, with the onset of scale invariance strengthening the analogy to the microscopic theory of critical phenomena. As is well known, that leads to the `$5/3$' spectrum, which was expected to be universal.

My own view is that it would be good to get it settled that the Kolmogorov spectrum holds for isotropic turbulence. There is still an absence of consensus about that. But the broader claim of universality has been supported by measurements of spectra in a vast variety of flow configurations; although, inevitably there have been instances where it is not supported. So we end up with yet another unresolved issue in turbulence. Is small-scale turbulence universal or not?

In order to consider whether or not the concept of symmetry could assist with this, it may be helpful to think in terms of definite examples. First, let us consider laminar flow in the x_1 direction between fixed parallel plates situated at $x_2=\pm a$. The velocity distribution between the plates will be a symmetric function of the variable x_2. If now we consider a flow where one plate is moving with respect to the other, and this is the only cause of fluid motion, then we have plane Couette flow and, as is well known, the velocity profile will now be an antisymmetric function of x_2. However, the molecular viscosity of the fluid will be unaffected by the different macroscopic symmetries and will be the same in both cases.

If we now extend this discussion to the case of turbulent mean velocities and inquire about the behaviour of the effective turbulent viscosity (ν_t, say: for a definition see Section 1.5 of reference [1]), it is clear that this will be very different in the two cases, and arguably that should apply to the cascade process as well.

In isotropic turbulence, the cascade is described by the Lin equation, with the key quantity being the transfer spectrum $T(k)$. Its extension to an inhomogeneous case will bring in a number of transfer spectra, such as T_{11}, T_{12} and so on. In order to cope with the dependence on spatial coordinates, the introduction of centroid and relative coordinates that we used in the previous post will prove useful. Recall that we considered a covariance function $C(\mathbf{x},\mathbf{x'})$, leaving the time variables out for simplicity and introduced the change of variables to centroid and relative coordinates, thus: $$\mathbf{R} = (\mathbf{x} + \mathbf{x'})/2 \qquad \mbox{and} \qquad \mathbf{r} = (\mathbf{x} - \mathbf{x'}).$$ In this case one component of the spectral tensor could be written as: $T_{11}(\mathbf{k}, R_2)$, where we have Fourier transformed with respect to the relative coordinate only. Then, at least in the core region of the flow, we could expand out the dependence on the centroid coordinate in Taylor series. In this way we could separate the wavenumber cascade from spatial effects, such as production and spatial energy transfer.

Ideally one could even use a closure theory: the covariance equation of the DIA has been validated by the LET theory [2] and, although some work has been done on this in the past, a really serious approach would require a lot of bright young people to get involved. Unfortunately, vast numbers of bright young people all over the world are involved in complicated pedagogical exercises in cosmology, particle theory, string theory, quantum gravity and so on, most of which has gone beyond any proper theoretical foundation. Ah well, important but less glamorous problems like turbulence must await their turn.

For completeness, I should emphasise that all flows discussed above are assumed to be incompressible and well-developed.

[1] W. D. McComb. The Physics of Fluid Turbulence. Oxford University Press, 1990.
[2] W. D. McComb and S. R. Yoffe. A formal derivation of the local energy transfer (LET) theory of homogeneous turbulence. J. Phys. A: Math. Theor. 50:375501, 2017.