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Up until about 1970, fundamental work on turbulence was dominated by the study of the energy spectrum, and most work was carried out in wavenumber space. In 1963 Uberoi measured the time-derivative of the energy spectrum and also the dissipation spectrum, in grid turbulence; and used the Lin equation to obtain the form of the transfer spectrum $T(k)$ [1]. Later on, this work was extended and refined by van Atta and Chen, who obtained the transfer spectrum more directly from the third-order correlation [2]. This seems to have been the peak of experimental interest in spectra, and from then on there was a growing concentration on the behaviour of the moments (strictly speaking, in the form of structure functions) in real space [3], [4].

Introducing the structure function of order n by $$S_n(r) = \langle \delta u_L^n(r) \rangle,$$ where $\delta u_L^n(r)$ is the longitudinal velocity difference, taken over a distance r, it is well known that, on dimensional grounds, they are expected to take the form $$S_n=C_n \,(\varepsilon r)^{n/3},$$ whereas investigations like [3] and [4] (and many following them over the years) found deviations from this that increased with order n. Such results gave increased traction to belief in intermittency corrections and anomalous exponents.

Yet, when one considers it, the moments of a distribution are equivalent to the distribution itself. It is well known that the moments are related to the distribution through its characteristic function which is its Fourier transform. From the simple example on page 529 of reference [5], we see that the characteristic function can be expanded out in terms of the moments. Hence the distribution can be recovered to any desired order from the infinite set of its moments. Therefore, when one measures moments to some order, one is merely assessing the accuracy with which one has measured the distribution itself. A plot of the measured exponent ζ_n against order n is no more or less than a plot of systematic experimental error. A glance at the plots of measured distributions in both [3] and [4] will make this point with compelling force, especially when one considers the wings of the distribution.

A brief overview of this topic and a number of more recent references may be found in [6]. Note that in that reference, a standard laboratory method of reducing systematic error was used to measure ζ_2 and showed that it tended towards the canonical value of $2/3$ as the Reynolds number was increased. As a matter of some slight interest, I learnt that method when I was about sixteen years old at school.

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