How big is infinity?

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In physics it is usual to derive theories of macroscopic systems by taking an infinite limit. This could be the continuum limit or the thermodynamic limit. Or, in the theory of critical phenomena, the signal of a nontrivial fixed point is that the correlation length becomes infinite. Of course, what we mean by `infinity' is actually just a very large number. But the mathematicians do not like this. In reference [1] below, the author states: '… statistical-mechanical theories of phase transitions tell us that phase transitions only occur in infinite systems'. She sees this as paradoxical because, as we all know, in everyday life we are surrounded by finite systems undergoing phase transitions. She further believes that the paradox can be resolved by working with constructive mathematics, rather than classical mathematics, which is what we all normally use.

My quotation from [1] is certainly open to deconstruction, and I doubt if many physicists would agree with it. What originally drew my attention to this particular problem is the situation in turbulence theory. As the Reynolds number is increased (or, the viscosity is decreased), the dissipation rate becomes independent of the viscosity. Physicists attribute this to the energy transfer by the nonlinear term in the equation of motion becoming scale-invariant. As the Reynolds number is increased even more, this scale-invariance extends further through wavenumber space, and nothing thereafter changes, either qualitatively or quantitatively. This in practical terms is the infinite Reynolds number limit, and it occurs at quite modest, finite values of the Reynolds number.

However, many mathematicians, harking back to a paper by Onsager [2] in 1949, believe that the infinite Reynolds number limit corresponds to zero viscosity; and, even more bizarrely, that the continuum properties of the fluid break down in this limit. Accordingly, they are driven to finding ways of making the Fourier representation of the inviscid Euler equation dissipative, by destroying its symmetry-based conservation properties. I have discussed this topic in three previous posts on 12, 19 and 26 November; and a paper, at that time in preparation, is now available on the arXiv as [3].

[1] Pauline van Wierst. The paradox of phase transitions in the light of constructive mathematics. Synthese, 196:1863, 2019.
[2] L. Onsager. Statistical Hydrodynamics. Nuovo Cim. Suppl., 6:279, 1949.
[3] W. D. McComb and S. R. Yoffe. The infinite Reynolds number limit and the quasi-dissipative anomaly. arXiv:2012.05614[physics.flu-dyn], 2020.