Here’s to mathematics and may it never be of use to anyone!

Here’s to mathematics and may it never be of use to anyone!

When I was a student, I read that mathematicians at conference dinners would drink a toast along the lines of the title of this piece. As an idealistic young man, I was quite shocked by this; and thought it very arrogant. Apart from anything else, it seemed to sell the entire discipline of applied maths very short indeed. I think that it took me until I was in my middle years to understand and indeed empathise with this statement.

In fact it can be seen as an indicator of what I call the *culture* of a subject. By `culture’ I mean something to do with a sense of what is the right way to think about physical problems, such as turbulence, or to attempt to solve them. The conviction that engineers, mathematicians and physicists have different cultures has grown on me over the years (and remember that I have been both mechanical engineer and theoretical physicist at different stages of my career).

A minor incident which helped my understanding of the mathematician’s attitude (or culture) happened when a colleague and I invigilated a class exam. A class from the maths department was being examined at the same time and their subject was something like `Functional analysis and Fourier analysis’. Well, I thought, this is something that I know a bit about. So I picked up a copy of the exam paper and was surprised to find that all the questions were to do with proving existence or uniqueness; not, as I would have expected, to actually work out some specific functional form when given certain initial conditions.

Another hint came at a workshop on turbulence at the Max Planck Institute for Mathematics in Bonn, sometime in the mid-1980s. All the speakers were theoretical physicists but a number of the resident mathematicians attended. When the first speaker had finished outlining his theory, one of the mathematicians said: `I would not dream of presenting such a very long calculation to an audience in one lecture.' That was a bit of a bummer as we on the physics side had thought that it was a theory, not a calculation. This chap, a rather flamboyant American, had his comeuppance later, when we all went to lunch at a pizza restaurant and he attempted to order with Italian intonations and theatrical gestures. The waiter was having none of it and pretended not to understand. So the flamboyant one had to calm down and order like the rest of us.

These and other encounters led me to understand that for mathematicians it is essential to be able to study those aspects of the subject which interest them, without constraints being imposed for any reason. And so it is for physics. For pure physics it is essential to be able to think the unthinkable (if necessary) and pursue curiosity based research. In passing, I should note that much physics research nowadays is really to be classed as applied physics. For instance, condensed matter physics, with its bedrock problems unsolved, seems to me to be very much materials science.

Naturally, it is in the subject of turbulence that these different cultures may clash. Theoretical physicists can publish in topics like particle theory, critical phenomena, cosmology or plasma physics, without having a mechanical engineer or applied mathematician refereeing the papers that they submit to journals. In turbulence, as I know from endless personal experience, this is not so. Of course this is exacerbated by the shortage of theorists working in the field, and even then there can be problems because of different agendas and an inability to put self-interest aside. I shall return to that particular aspect in a future blog, but for the moment I am concerned with the different cultures. Various instances can be found in the well-known lectures of Philip

Saffman [1].

These lectures, and a previous set, are opinionated and quite stimulating to read; not least, in my case, because I so often disagree with them. In reference [1] on page 294, Saffman has a section on statistical methods. He begins with the general statistical theories, as pioneered by Kraichnan and the following quotation is of interest:

`The techniques of the statistical theory are supposed to be rigorous and analytical. They are certainly impressive with their talk of Greens (sic) functions, propagators, diagrams, Galilean invariance, and other jargon of physics and probability theory, and the resulting integro-differential equations are sufficiently complicated to suspend belief, but in fact the approximations are just as ad hoc and lacking in mathematical justification as those of Reynolds stress modelling. Also, the absence of a physical basis is unfortunately usually combined with the obscurity of the details.'

In this Saffman certainly made his position clear. It perfectly underlines the existence of a culture clash. In fact a detailed deconstruction of that quotation (which would not be entirely unsympathetic to Saffman) could be of interest and I might return to that for a blog on its own later on. However, to bring this to a point, he ends up quoting Leslie (1973) and Bradshaw (1976) on the significance of Kraichnan's work but does not support his comments (which are rather confused) with any actual references to Kraichnan.

One point I should mention, is that he says that Kraichnan's theory `can *postdict* the Kolmogorov constant, which may not exist because of intermittency, …' In later years, at the NASA-ICASE workshop on turbulence in 1984, we discussed his use of the word `postdict' and he conceded that if a theory were genuinely from first principles it would be appropriate to say *predict*. Of course the question arises, was Kraichnan's theory genuinely from first principles? And that is where Saffman’s criticisms really have some force. Again, this is something that I shall return to in later posts.

[1] P. G. Saffman. Problems and progress in the theory of turbulence. In H. Fiedler, editor, Structure and Mechanisms of Turbulence II, volume 76 of Lecture Notes in Physics, pages 273-306. Springer-Verlag, 1977.