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When I was working for my PhD with Sam Edwards in the late 1960s, my second supervisor was David Leslie. We would meet up every so often to discuss progress, and I recall that David was invariably exasperated by our concentration on asymptotic behaviour at high wavenumbers. He was strongly motivated towards applications, and felt that the production process at low wavenumbers was more important. To him the dissipation was uninteresting. He used to say to us: `you are messing about down in the drains, when the interesting stuff is all in the production region.'

We had many good humoured arguments but none of us changed our positions. Yet with the passage of time, I increasingly feel that David had a point; even when we restrict our attention to isotropic turbulence. In fact, I would go further and argue that much of the confusion over the Kolmogorov (1941) picture, arises from a failure to see that the dissipation is not the primary quantity. And even when one arrives legitimately at the dissipation (having first considered the production and then the inertial transfer rates), there is often confusion between the instantaneous dissipation rate and the mean dissipation rate. I have myself contributed to that confusion, and this is an opportunity to set matters straight. But first let us consider what the usual practices are.

Kolmogorov used ϵ for the instantaneous dissipation and $\bar{\epsilon}$ for the mean. Then, in 1953, Batchelor used ϵ for the mean dissipation (see equation (6.3.2) in the second edition of his book). A few years later, in 1959, Hinze favoured ε for the mean dissipation, and this has tended to prevail ever since, particularly in theoretical physics, where ϵ is used as an expansion parameter: e.g. the famous ϵ-expansion!

In my 1990 book [1], I used ε for instantaneous dissipation in equation (1.17) and $\langle \varepsilon \rangle$ for the mean dissipation in equation (A18). Unfortunately, where I discuss the Kolmogorov variables, in Chapter Two and elsewhere, it is clear that I intend ε to be the mean dissipation rate. In fact this is the most prevalent usage throughout the literature, at least in theoretical work. When one thinks about it; well it makes sense. One is only ever really interested in mean quantities and a hat notation can be used for instantaneous values where they are required. In my later book [2], I tried to sort this out, as follows: \[\widehat{\varepsilon}=\; \mbox{instantaneous dissipation}\] \[\varepsilon=\;\mbox{mean dissipation}\] \[\varepsilon_D = -\ddt E :\;\mbox{the eddy decay rate}\] \[\varepsilon_T = \Pi_{max} :\;\mbox{maximum rate of inertial transfer}\] \[\varepsilon_W :\;\mbox{rate at which stirring forces do work on the fluid}\].

So how does this help us with Kolmogorov, back in 1941? Well, in fact it helps us with Obukhov who, unlike Kolmogorov, worked in wavenumber space where there actually is a turbulence cascade. Obukhov realised that as the Reynolds number increased, there would be a limit where the inertial transfer rate became equal to the dissipation. As the Reynolds number continued to increase, this region of maximum energy transfer would increase in extent, to ever higher wavenumbers. This behaviour has been amply confirmed and is an example of scale invariance. It was recognized by both Obukhov and Onsager that in this range of wavenumbers the spectrum would take the form \[E(k) \sim \varepsilon_T^{2/3}k^{-5/3}.\] If you wish, you can replace the rate of inertial transfer with the dissipation rate. If you want to derive Kolmogorov’s $r^{2/3}$ law, then just Fourier transform the Obukhov result for the spectrum. It is the form that has been derived by a properly formulated physical argument. It would be difficult to see how anyone could drag in the so-called intermittency corrections!

[1] W. D. McComb. The Physics of Fluid Turbulence. Oxford University Press, 1990.
[2] W. David McComb. Homogeneous, Isotropic Turbulence: Phenomenology, Renormalization and Statistical Closures. Oxford University Press, 2014.