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Vacation post No 1. I will be out of the virtual office until Monday 31 August.
When I used to lecture final-year undergraduates in mathematical physics, there were often quite a few mathematicians attending and I would sometimes tease them by pointing out that mathematicians try to prove the ergodic theorem whereas physicists don’t need to. We know it must be true! This was always taken in good part, but it wasn’t really a joke, because I believe it to be literally true. Progress in physics from earliest times has proceeded from experimental observation, which is then codified in mathematical theory. When a new observation arises and does not agree with the existing theory, then so much the worse for the theory. We have to devise a new and better one. (I believe the Hegelian position is the exact opposite of this: so much the worse for the observation!)

The only exception to this that I know of is the work of the great Paul Dirac, who actually started his working life as an electrical engineer and only later qualified in mathematics. He tackled the problem of deducing a relativistic form of the Schrödinger by purely mathematical methods and ended up predicting the existence of antimatter. Nice one Paul!

If one is going to have an exception, what an exception to have. The only thing that I can think of which might be comparable, is the work of Emmy Noether. Her theorem that continuous symmetry of a physical quantity implies its conservation underpins the whole of fundamental theoretical physics. And of course much mathematical work has gone into the development of modern formulations from the original observation-based forms, such as Newton’s laws of motion.

However, I don’t know enough about Noether’s theorem to be sure about whether or not it also represents a significant exception. I still intend to rectify this, although I have been intending to do so, for many years.

As regards the relevance of my original question to turbulence, I can come up with a specific example in a related field. A few years before I retired, I had some discussions with a mathematician about problems in soft (condensed) matter. This arose in a social way, in that one of my colleagues had attended a party in the maths department and got talking to a young mathematician who bemoaned the fact that he had no one to discuss his work with. My colleague knew that I had published something in this area [1] and suggested that we make contact. As a result we had a number of discussions (and some games of badminton!) and it was clear that we were poles apart in the way we looked at things. Nevertheless, one specific point emerged. He had reservations about the (at that time) famous KPZ equation for nonlinear deposition. On purely mathematical grounds (something to do with simultaneously working with generalized functions and Fourier transforms, I think) he had concluded that the KPZ equation was mathematically unsound and needed a counter-term to be added to deal with this. Accordingly he was quite surprised to find that my co-author and I had already come to this conclusion on purely physical grounds and that we had identified the requisite term to be added [1].

It seems to me that modern theoretical physics is dominated by this sort of pure mathematical approach which may in fact be sterile without a new physical hypothesis of the kind that physicists can actually understand to be such. In the rather humbler discipline of turbulence theory, I note many papers which seem to be predicated on the assumption that one must take account of singularities. I believe this activity may actually be harmful, as well as unnecessary, because it makes people unsure about things. For example, when a referee insists that I qualify some statement about taking a limit or making an expansion, with the phrase `provided that no singularity occurs' I feel that I am being forced to make use of the mathematician's comfort blanket. Frankly, I would rather rely on the physicist's comfort blanket, which is based on the interlocking physical picture which in turn is based primarily on observation. Just bear it in mind: we physicists know that the ergodic theorem holds.

[1] W. D. McComb and R. V. R. Pandya. Hidden symmetry in a conservative equation for nonlinear growth. J. Phys. A: Math. Gen., 29:L629, 1996.