Further thoughts on free decay of isotropic turbulence.

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In the previous post I discussed the initial value problem posed by the free decay of the energy in isotropic turbulence, along with things that we ought to bear in mind when considering its experimental or DNS realisations. We should also mention the more general problem of the free decay of two-point covariances (or spectra) as that merits a few words in the context of both DNS and the study of two-point statistical closures. However, before considering it, we should first consider an outstanding question about the simpler case: at what stage is the turbulence to be considered as evolved?

The question arises because the initial state of the turbulence is not actually a solution (or, more accurately, derived from a solution) of the Navier-Stokes equation. For the purely mathematical problem, we may indeed assume that the initial field corresponds to isotropic turbulence. But for grid turbulence, the wakes that form behind the bars of the grid are expected to coalesce into a three-dimensional turbulent field, which dies away with downstream distance. This stationary stream-wise decay has to be converted to decay with time by invoking Taylor's hypothesis, but the crucial question is: at what distance downstream can the turbulence be said to be evolved?

The same question must arise with DNS, where we specify an initial spectrum on a lattice. Such initial spectra are arbitrarily chosen to have suitable properties. In particular, they are chosen to be peaked at low values of wavenumber, so that the evolution of turbulence can be seen as the spectrum not only decreases in magnitude, but also spreads out, as time goes on. So once again we wish to know at what time the spectrum will be representative of turbulence, rather than the initial conditions.

Probably most investigations into this topic have been concerned with establishing whether or not the decay follows a power law; and, if so, what that power law is. In fact some researchers cite the onset of power-law behaviour as indicating that the turbulence is well developed. Yet there is at least once situation where the need for a definite criterion matters and this is the study of the dimensionless dissipation in terms of its dependence on the Reynolds number. This is known to follow a characteristic curve in which it asymptotes to a constant value with increasing Reynolds number.

Now, for stationary turbulence, the existence of a unique curve is unambiguous on both experimental (i.e. DNS) and theoretical grounds), but for free decay there is a fair amount of scatter between the various investigations. When we began working on this problem at Edinburgh some years ago, we were surprised to find that most researchers seemed rather vague about the stage of the decay process at which their measurements were taken. It seemed to us that this was likely to prove crucial. An investigation would consist of carrying out a free decay simulation at a particular Reynolds number; then repeating it for a higher Reynolds number, and so on. Then the problem at any Reynolds number is to choose a decay time to take measurements that corresponds in some sense to `the same stage' at other Reynolds numbers. This is not a trivial problem and we decided to look into it in detail [1].

When a turbulence simulation is started from an arbitrary initial velocity field with a Gaussian distribution, both the inertial transfer and the skewness grow from zero, pass through a peak and then decay. In contrast, the dissipation rate starts with a finite value and either decays (low

Reynolds numbers, say $R_\lambda(0) \leq 25$) or rises to a peak and then decays (higher Reynolds numbers). The existence of a peak offers the possibility of a well-defined criterion which would allow the results of one investigation to be compared with another. We plotted graphs of dimensionless dissipation $C_\varepsilon (t_e)$ against Reynolds number for various choices of evolved time t_e (see Fig. 13) and found that the resulting behaviour depended strongly on the choice made. For instance, choosing t_e to be either based on the peak skewness or peak inertial transfer led to the curve tending to zero. As there is no peak dissipation for low Reynolds numbers (and the variation of C_ε is predominantly a low Reynolds number phenomenon) this appeared to rule peak dissipation out as a criterion. However, we found that a composite criterion, based on peak transfer at low Reynolds numbers and on peak dissipation at larger Reynolds numbers, where a peak existed, gave very interesting results, with the dimensionless dissipation curve being very like the stationary forced case, and tending to a value of about 0.5.

I do not claim that these results are prescriptive or definitive in any way, although they are certainly quite plausible. But I hope they will encourage others to investigate further. If this is not done, the studies in decaying turbulence will remain a hodge-podge where variations between investigations are often probably due to a failure to compare like with like.

Lastly, in my previous post I said that at an early stage in my career I resolved to stay clear of the problem of free decay. To avoid any appearance of inconsistency I should point out that this resolution was limited to the theoretical problem of predicting the decay rate of the energy. In the late 1970s we began studying the LES theory applied to the problem of free decay of two-point, two-time statistics. This work was reported in 1984 [2], and involved a detailed comparison with DIA, using the same initial spectra and computational methods as previously used by Kraichnan. This allowed `like for like' comparisons in great detail, which was still the case when the comparisons were extended to DNS in later years. So the onset problem did not, as such, arise.

[1] S. R. Yoffe and W. D. McComb. Onset criteria for freely decaying turbulence. Phys. Rev. Fluids, 3:104605, 2018.
[2] W. D. McComb and V. Shanmugasundaram. Numerical calculations of decaying isotropic turbulence using the LET theory. J. Fluid Mech., 143:95-123, 1984.