Is there an alternative infinite Reynolds number limit?

Is there an alternative infinite Reynolds number limit?

I first became conscious of the term *dissipation anomaly* in January 2006, at a summer school, where the lecturer preceding me laid heavy emphasis on the term, drawing an analogy with the concept of anomaly in quantum field theory, as he did so. It seemed that this had become a popular name for the fact that turbulence possesses a finite rate of dissipation in the limit as the viscosity tends to zero. I found the term puzzling, as this behaviour seemed perfectly natural to me. At the time it occurred to me that it probably depended on how you had first met turbulence, whether the use of this term seemed natural or not. In my case, I had met turbulence in the form of shear flows, long before I had been introduced to the study of isotropic turbulence in my PhD project.

Back in the real world, the experiments of Osborne Reynolds were conducted on pipe flow in the late 1890s, and this line of work was continued in the 1930s and 1950s by (for example) Nikuradse and Laufer [1]. This led to a picture where turbulence was seen as possessing its own resistance to flow. The disorderly eddying motions were perceived to have a randomizing effect analogous to, but much stronger than, the effects of the fluid’s molecular viscosity. This in turn led to the useful but limited concept of the *eddy viscosity*. As the Reynolds number was increased, the eddy viscosity became dominant, typically being two orders of magnitude greater than the fluid viscosity.

In principle, there are three alternative ways of varying the Reynolds number in pipe flow, but in practice it is just a matter of turning up the pump speed. Certainly no one would try to do it by decreasing the viscosity or increasing the pipe diameter. In isotropic turbulence, the situation is not so straightforward, as we use forms of the Reynolds number which depend on internal length and velocity scales. Indeed the only unambiguous characteristic which is known initially is the fluid viscosity.

An ingenious way round this was given by Batchelor (see pp 106 – 107, in [2]), who introduced a Reynolds number for an individual degree for freedom (i.e. wave-number mode) as $R(k) = [E(k)]^{1/2}/\nu k^{1/2}$, in terms of the wavenumber spectrum, the viscosity and the wave-number of that particular degree of freedom. He argued that the effect of decreasing the viscosity would be to increase the dominance of the inertial forces on that particular mode, so that the region of wave-number space which is significantly affected by viscous forces moves out towards $k=\infty$. He concluded: `In the limit of infinite Reynolds number the sink of energy is displaced to infinity and the influence of viscous forces is negligible for wave-numbers of finite magnitude.’ A similar conclusion was reached by Edwards from a consideration of the Kolmogorov dissipation wave-number [1], who showed that the sink of energy at infinity could be represented by a Dirac delta function.

It is perhaps also worth mentioning that the use of this local (in wave-number) Reynolds number provides a strength parameter for the consideration of isotropic turbulence as an analogous quantum field theory [3].
Evidently the conclusion that the infinite Reynolds limit in isotropic turbulence corresponds to a sink of energy at infinity in k-space seems to be well justified. Nevertheless, this use of the value infinity in the mathematical sense is only justified in theoretical continuum mechanics. In reality it cannot correspond to zero viscosity. It can be shown quite easily from the phenomenology of the

subject that the infinite Reynolds number behaviour of isotropic turbulence can be demonstrated asymptotically to any required accuracy without the need for zero viscosity. We shall return to this in a later post.

1. W. D. McComb. The Physics of Fluid Turbulence. Oxford University Press, 1990.
2 G. K. Batchelor. The theory of homogeneous turbulence. Cambridge University Press, Cambridge, 1st edition, 1953.
3. W. David McComb. Homogeneous, Isotropic Turbulence: Phenomenology, Renormalization and Statistical Closures. Oxford University Press, 2014.