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Shortly after I retired, I began a two-year travel fellowship, with the hope of having interesting discussions on various aspects of turbulence. I’m sure that I had many interesting discussions, particularly in trying out some new and half-baked ideas that I had about that time, but what really sticks in my mind are certain unsatisfactory discussions.

To set the scene, I had recently become aware of Lundgren’s (2002) paper [1] and, having worked through it in detail, I was convinced that it offered a proof that the second-order structure function took the Kolmogorov `2/3’ form asymptotically in the limit of infinite Reynolds numbers. There is of course little or no disagreement about Kolmogorov’s derivation of the `4/5’ law for the third-order structure function. For stationary turbulence, it is undoubtedly asymptotically correct in the infinite Reynolds number limit. But in order to find the second-order form, Kolmogorov had to make the additional assumption that the skewness of the longitudinal derivative became constant in the infinite Reynolds number limit. Introducing the skewness S as $S=S_3(r)/S_2(r)^{3/2}$, and substituting the `4/5’ law for S_3, results in the well-known form $S_2(r)=(-4/5S)^{2/3}\varepsilon^{2/3}r^{2/3}\equiv C_2\varepsilon^{2/3}r^{2/3}$. Numerical results do indeed suggest that the skewness becomes independent of the Reynolds number as the latter increases, but it remains a weakness of the theory that this assumption is needed.

Lundgren [1] started, like Kolmogorov, from the Karman-Howarth equation (KHE), and did the following. He put the KHE in dimensionless form by a generic change of variables based on time-dependent length and velocity scales, l and u. He then chose to examine: first, Von Karman scaling; and secondly, Kolmogorov scaling, with appropriate choices for l and u. In both cases, he solved for the scaled second-order structure function by a perturbation expansion in inverse powers of the Reynolds number. He then employed the method of matched asymptotic expansions which recovered the Kolmogorov form for S_2. The `4/5’ law was also recovered for S_3, both results naturally following in the large Reynolds number limit. A more extensive account of this work can be found in Section 6.4.6 of my 2014 book.

Before setting off on my travels, I consulted a colleague who, although specializing in soft matter, had some familiarity with turbulence. To my surprise he seemed quite unenthusiastic about this work. He said something to the effect that it was a pity that Lundgren had to assume the same scaled form for both the second-order and the third-order structure functions. Now, on reflection I saw that this was nonsense. All Lundgren did was introduce a change of variables: this is not an assumption; it merely restates the problem, as it were. Secondly, the basic Kolmogorov theory deals with the probability distribution functional, and this means that all the moments (and hence structure functions) will be affected in the same way by any operation on it [2].

On the first of my visits, I began to discuss this with Professor X, who seemed very sceptical at first, then his comments seemed increasingly irrelevant, then he realised that he was thinking of an entirely later piece of work by Lundgren. At that point the discussion fizzled out.

On a later visit to a different university, at an early stage in the discussion with Professor Y, I commented that the method relied on the fact that the Karman-Howarth equation was local in the variable r. To which he swiftly replied: `Yes Tom does have to assume that.' That effectively brought things to a close, because once again we are faced with nonsense. In fact this particular individual seems to believe that the existence of an energy cascade implies that the KHE is nonlocal! But of course the nonlocalness is confined to the Lin equation in wavenumber space.

On a later occasion, I tried to bring the subject up again, but no luck. He said: `Tom just makes the same assumptions as Kolmogorov did. So there is nothing new.' At this point I finally gave up. However, as we have just seen, Kolmogorov has to assume that the skewness S becomes constant as the Reynolds number increases. In contrast, the Lundgren analysis actually shows that this is so. In addition, it also provides a way of assessing systematic corrections to the `4/5' law at large but finite Reynolds numbers.

The basic theoretical problems in turbulence are very hard and perhaps even impossible to solve, in a strict sense. However, the fact that lesser problems of phenomenology are plagued by controversy, with issues remaining unresolved for decades, seems to me to be a matter of attitude (and culture) that leads to a basic lack of scholarship. I think we need to trade in the old turbulence community and get a new one.

[1]Thomas S. Lundgren. Kolmogorov two-thirds law by matched asymptotic expansion. *Phys. Fluids*, 14:638, 2002.

[2] I have to own up to an error here. For years I argued that only the second- and third-order structure functions were involved in Kolmogorov and hence conclusions based on higher-order moments were irrelevant. Then (quite recently!) I noticed in a paper by Batchelor the comment that as the hypotheses were for the pdf, they automatically applied to moments of all orders.