Symmetry and supersymmetry

University of Edinburgh PG Colloquium

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Maxwell Institute Graduate School,
University of Edinburgh

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Figure: James Clerk Maxwell (Wikimedia Commons).

James Clerk Maxwell

- Mathematical physicist
- 1831-1879
- Born in Edinburgh
- Edinburgh & Cambridge Universities

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  - unification of electromagnetism
  - description of light as electromagnetic wave

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Figure: Statue of Maxwell, Edinburgh (Wikimedia Commons).

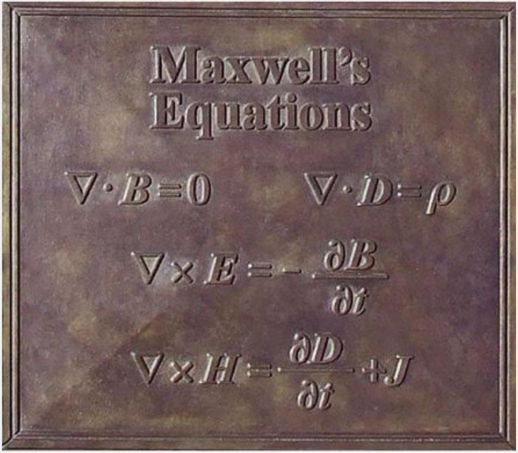

Figure: Plaque, Edinburgh (Wikimedia Commons).

Plan

Maxwell's equations

Maxwell's Equations

$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\epsilon_0}$	Gauss (electrical)
$\nabla \cdot \vec{\mathbf{B}} = 0$	Gauss (magnetic)
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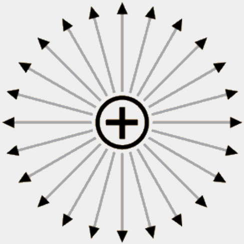

Figure: Electric fields for a point charge (Hyperphysics).

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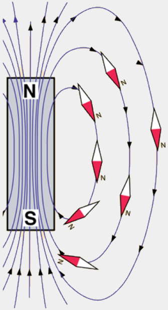

Figure: Magnetic fields (Hyperphysics).

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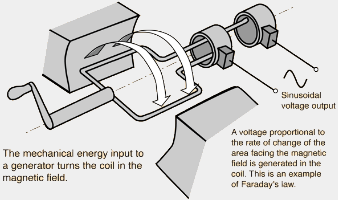

Figure: An AC generator (Hyperphysics).

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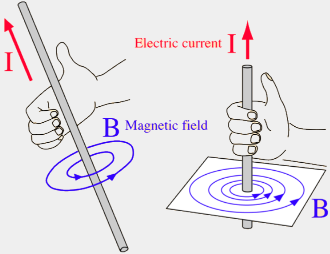

Figure: A solenoid (Hyperphysics).

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$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{\mathbf{J}}$ Continuity

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But this is a problem for 19th century physics…

Symmetries in classical physics

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- Example: N point particles in Newtonian gravity, $i = 1, \ldots, N$

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- Invariant under $\vec{\mathbf{x}}_i \to \vec{\mathbf{x}}_i + \vec{\mathbf{a}}$

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- Simplify problems
- Noether's theorem: continuous symmetries are associated with conservation laws
  - Example: momentum conservation
- Provide principles for building theories

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Physical states might not share the symmetries of the theory

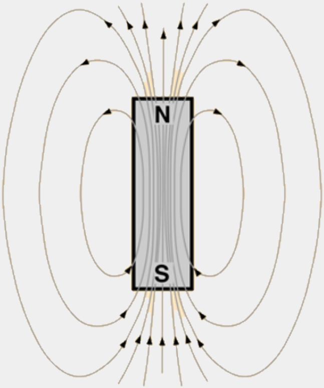

Figure: A bar magnet(Hyperphysics).

Symmetries: Theories vs solutions

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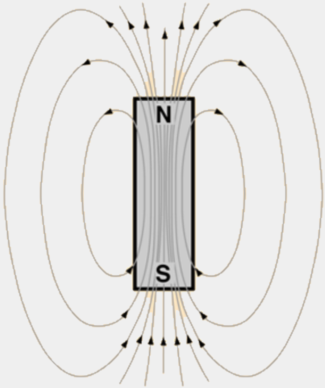

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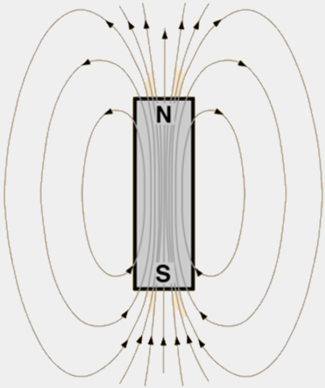

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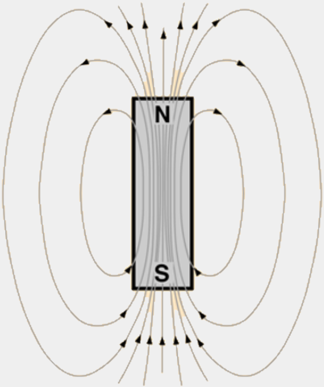
- Broken symmetry maps to a different physical state



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Maxwell's equations are not invariant under Galilean boosts!

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But these aren't all of the symmetries of Maxwell's equations...

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Gauge symmetry actually forces electromagnetism onto us!

Symmetries in Quantum Field Theory

Dirac equation:

$$mc^2\beta\psi - i\hbar c\sum_{i=1}^{3}\alpha_i\frac{\partial\psi}{\partial x_i} = i\hbar\frac{\partial\psi}{\partial t}$$

where $\psi : \mathbb{R}\times\mathbb{R}^3 \to \mathbb{C}^4$ and $\alpha_i, \beta \in M_{4\times 4}(\mathbb{C})$.

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- prediction of the existence of anti-matter

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- The first quantum field theory: *quantum electrodynamics*

The Standard Model and beyond

The Standard Model of Particle Physics

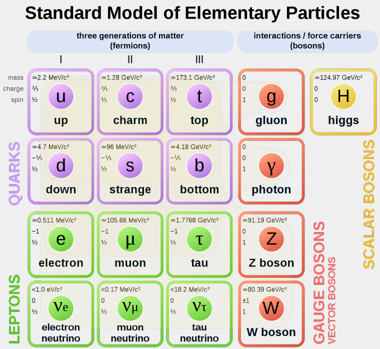
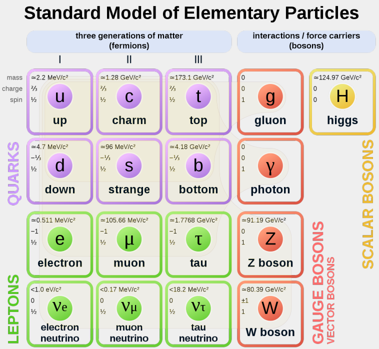


Figure: SM particles (Wikimedia Commons).



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Based the symmetry principles already mentioned

- Special relativity
- (Non-abelian) gauge symmetries

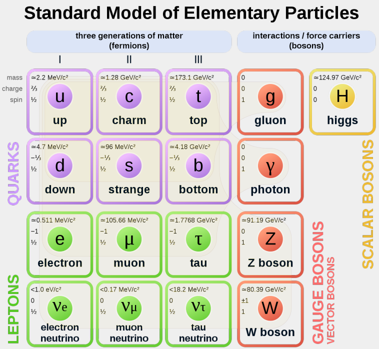

Figure: SM particles (Wikimedia Commons).

Based the symmetry principles already mentioned

- Special relativity
- (Non-abelian) gauge symmetries
  - Organise particles

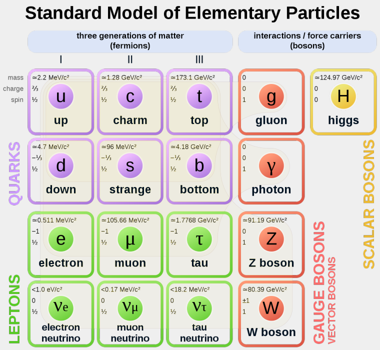

Figure: SM particles (Wikimedia Commons).

Based the symmetry principles already mentioned

- Special relativity
- (Non-abelian) gauge symmetries
  - Organise particles
  - Constrain interactions

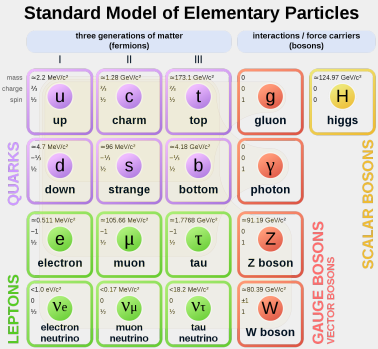

Figure: SM particles (Wikimedia Commons).

Based the symmetry principles already mentioned

- Special relativity
- (Non-abelian) gauge symmetries
  - Organise particles
  - Constrain interactions
  - Predicted new particles

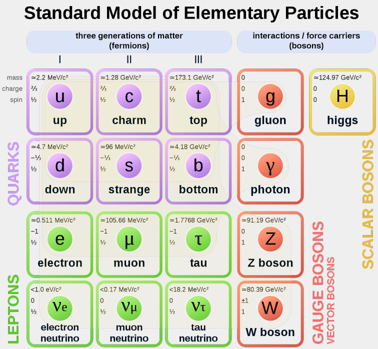

Figure: SM particles (Wikimedia Commons).

The SM is extremely successful:

- Constituents of ordinary matter
- Microscopic forces
- All particle collider experiments to date

It has some serious issues though:

- No accounting for "dark matter" or "dark energy"
- Hierarchy problem
- Issues with particle masses
- Other technical issues...

Supersymmetry

Possible solution to issues with SM: a new symmetry!

Supersymmetry

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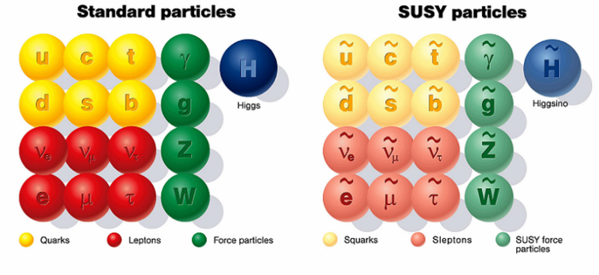

Figure: MSSM Particles (DESY).

- Symmetry between fermions and bosons
- Every particle has a "superpartner" of the opposite type
- Constrains interactions between particles

Trouble for supersymmetry

Recent experimental results rule out previously favoured supersymmetric models

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Recent experimental results rule out previously favoured supersymmetric models

- Supersymmetry not dead, but treated with much more skepticism now
- More ideas being explored, many based on symmetry considerations

Have we followed the wrong path?

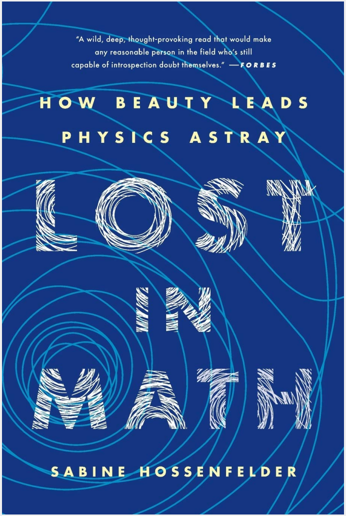
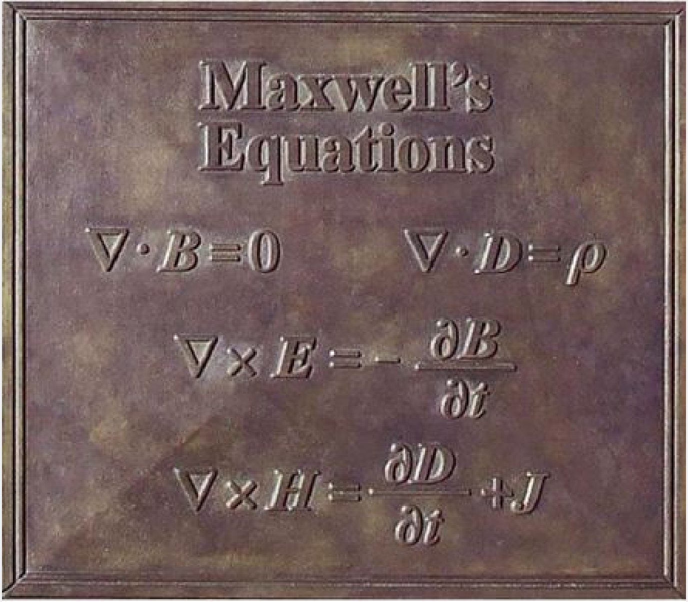


Figure: Lost in Math by Sabine Hossenfelder.


Maxwell's
Equations
$\nabla \cdot B = 0$
$\nabla \cdot D = \rho$
$\nabla \times E = -\frac{\partial B}{\partial t}$
$\nabla \times H = \frac{\partial D}{\partial t} + J$