FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

NATURWISSENSCHAFTLICHE FAKULTÄT

A Mathematical Challenge: Bookings and Technical Capacities in the European Entry-Exit Gas Market

Johannes Thürauf

Discrete Optimization, FAU Erlangen-Nürnberg

Energy Seminar Edinburgh, May 7 2020

DISCRETE OPTIMIZATION

The European Entry-Exit Gas Market System

Perfect Competition

Multilevel Entry-Exit Gas Market Model

A four level model for the European Entry-Exit Gas Market[1]:

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- Level 4: Cost-optimal transport through the network by the TSO

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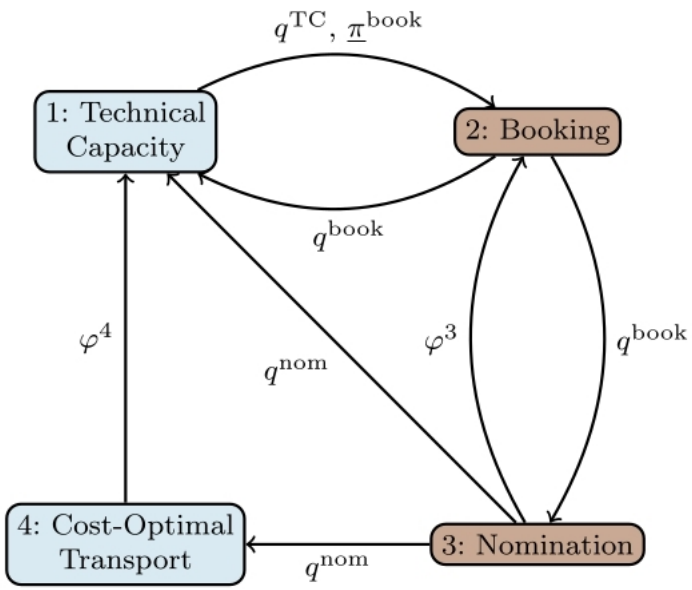

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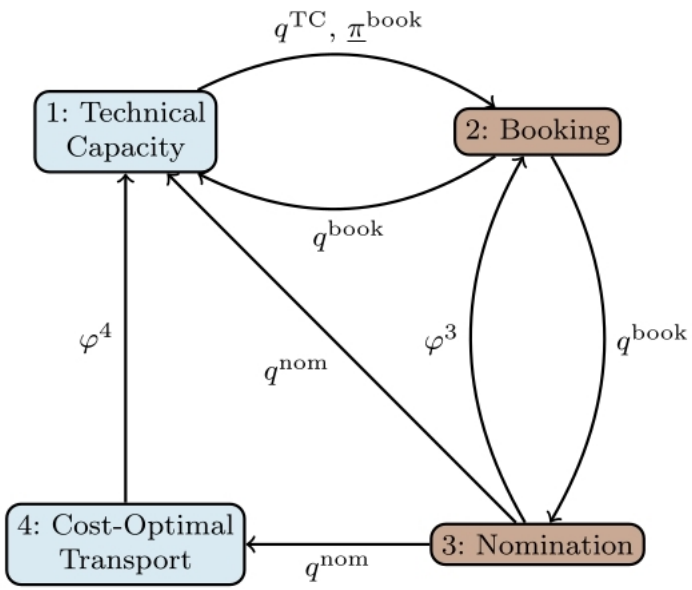

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Main Goal: Decouple trading and transport using technical capacities

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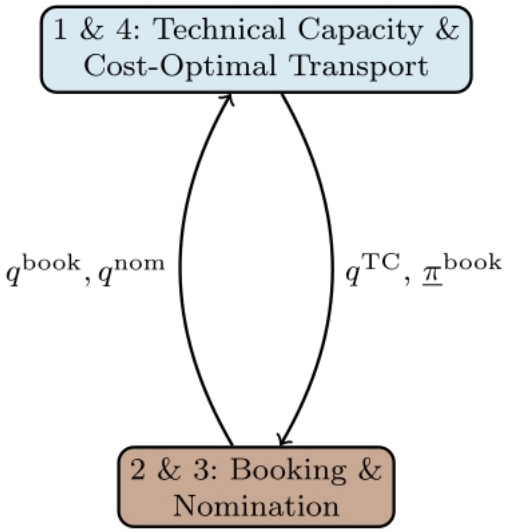
Reformulation: Bilevel Model

Structure of Bilevel Model

Upper Level (TSO):



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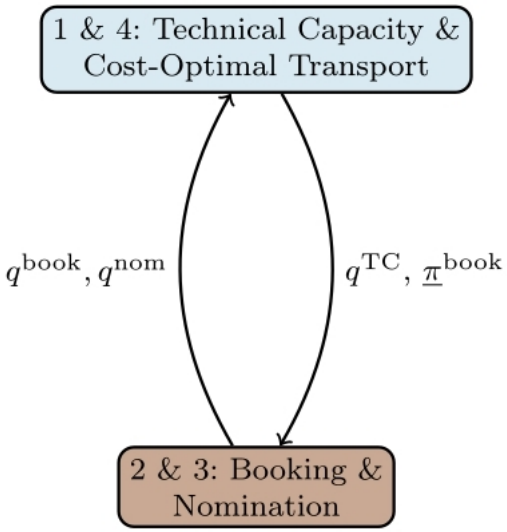

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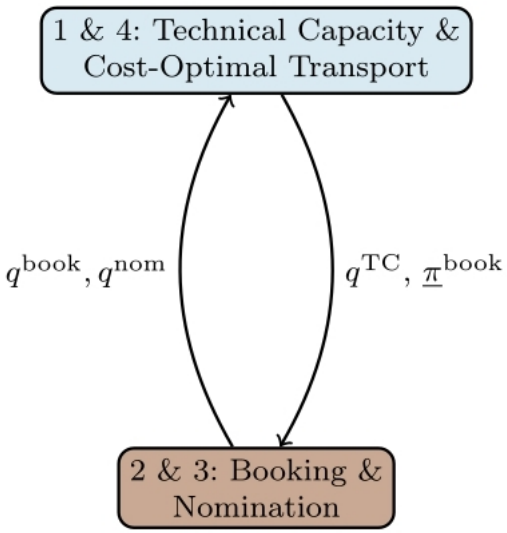

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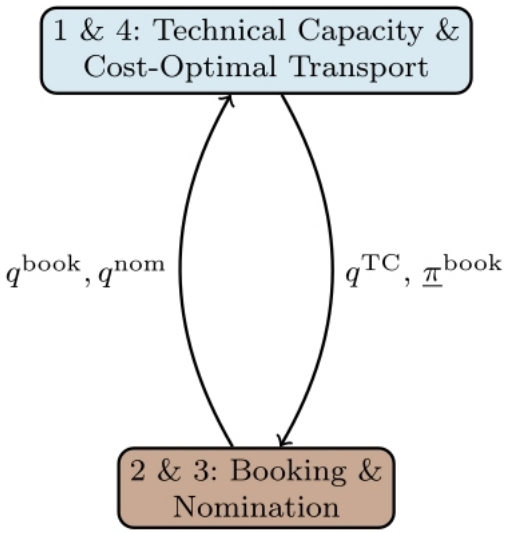

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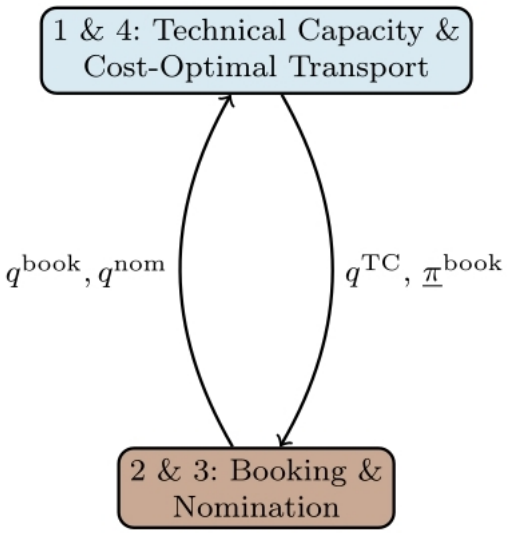

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Main Challenge: technical capacities

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is feasible.

No standard techniques of robust optimization are applicable

Tree-Shaped Networks

Joint Work Robinius, Schewe, Schmidt, Stolten, and Welder[2]

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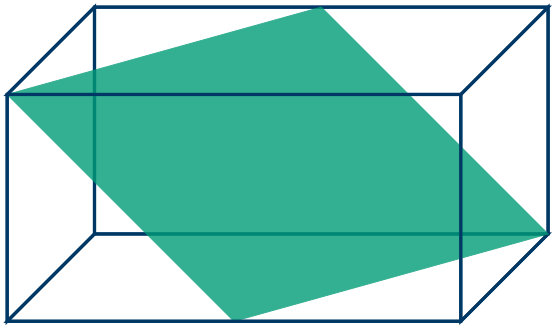

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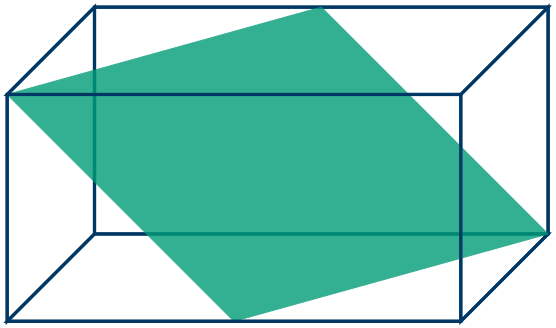

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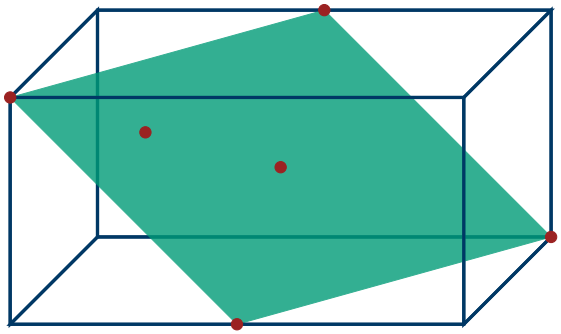

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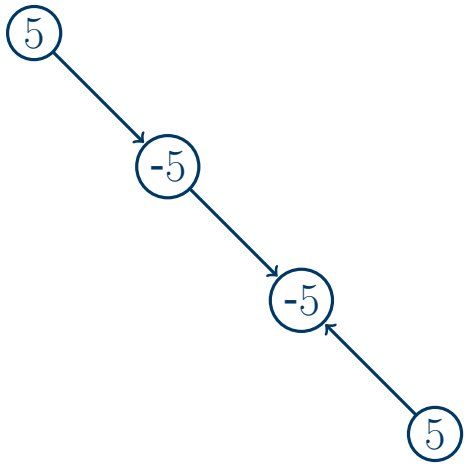

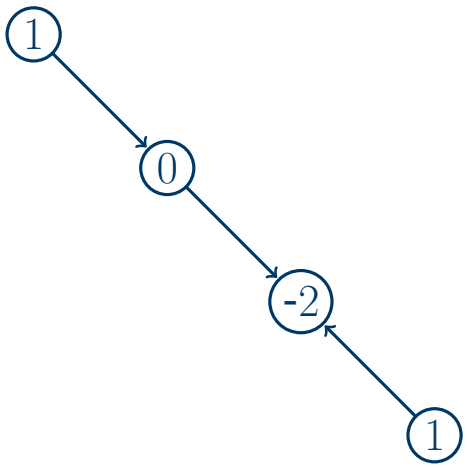

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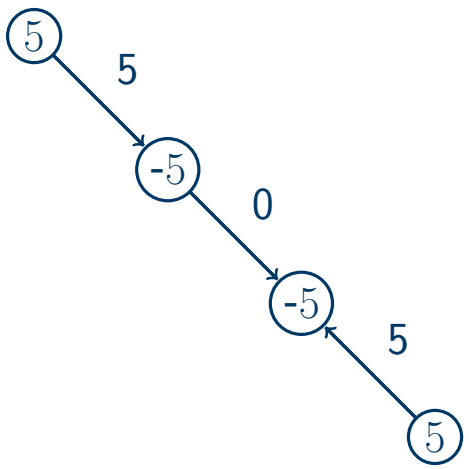

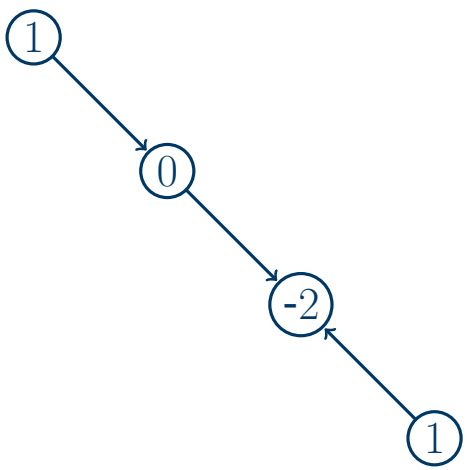

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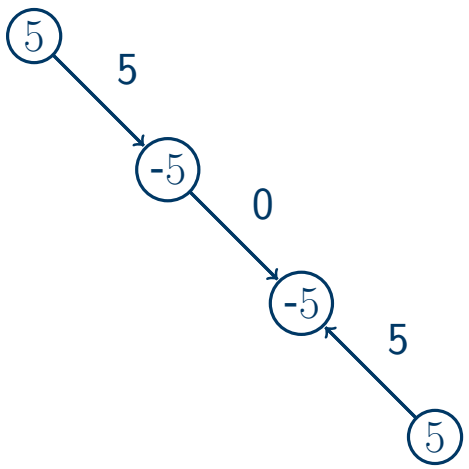

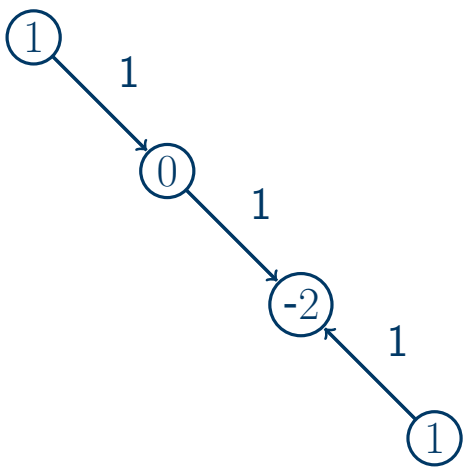

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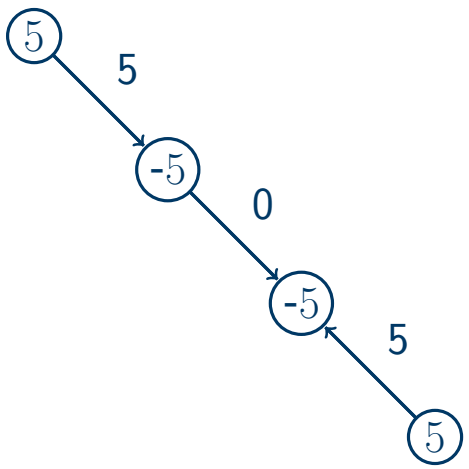

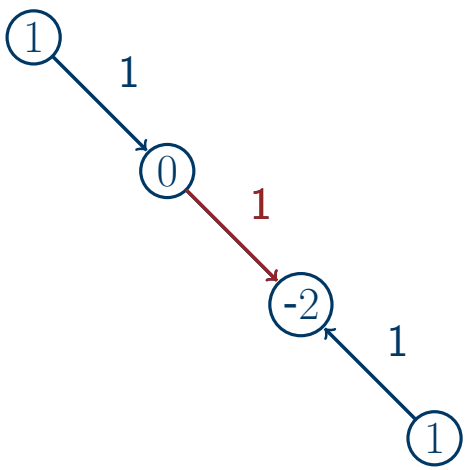

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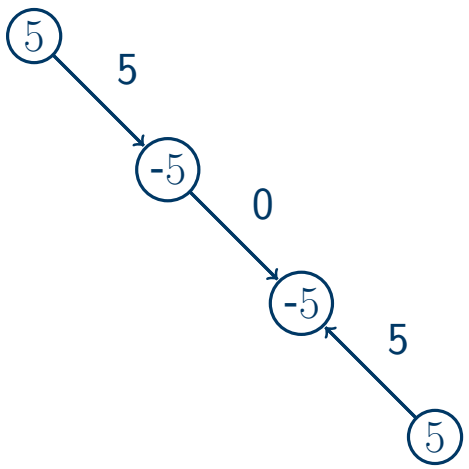

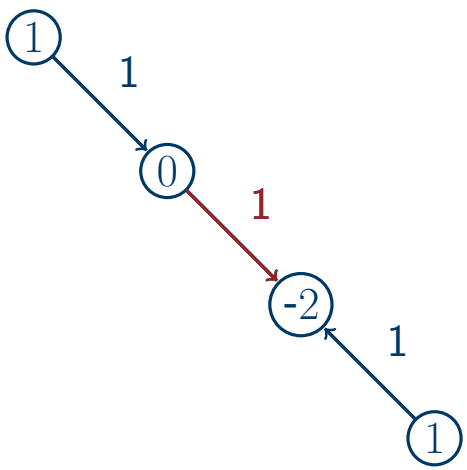

→ Greedy Approach does not lead to a robust solution

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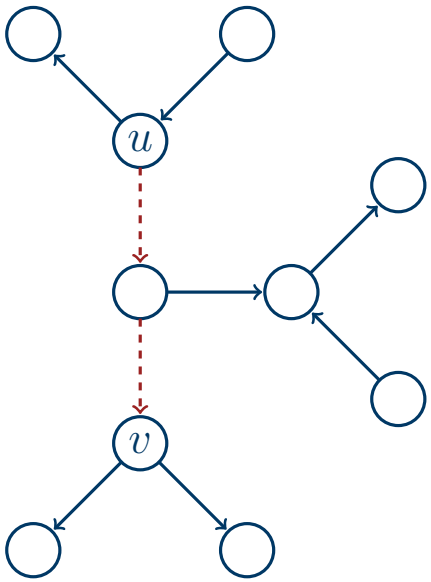
Technical capacities are feasible if and only if these special nominations are feasible.

Structure of Special Nominations

Special Nominations

Consider entry u and exit v.



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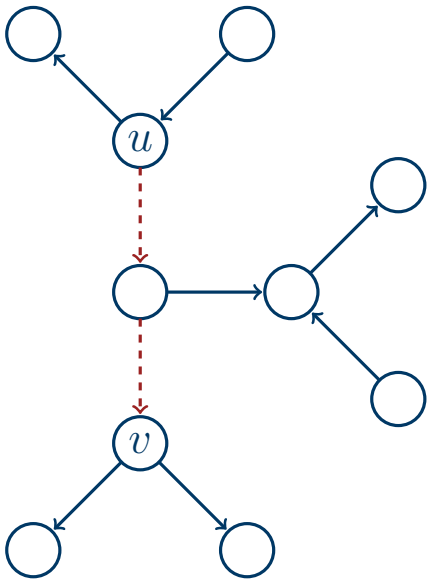
Special nomination $q^{\text{nom}} \in B_{u,v} \subseteq B$ and its unique flows q satisfy:

(1) Node u supplies node v.



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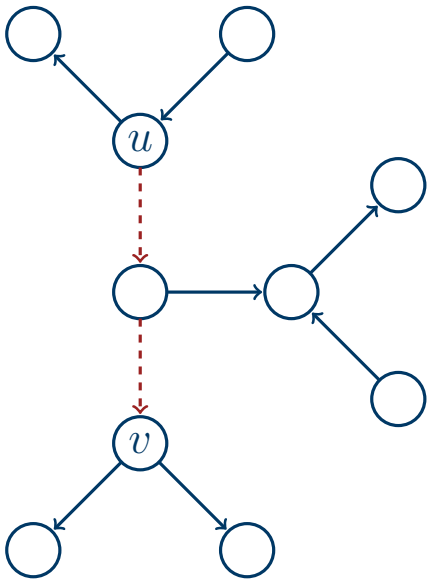

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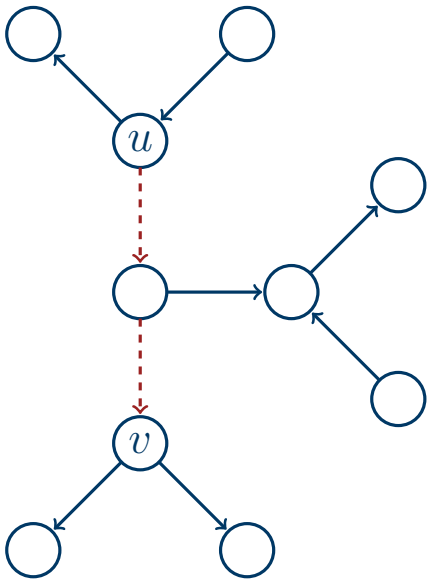

Lemma (Existence of Special Nominations)

For each entry u and exit v a special nomination exists: $B_{u,v} \neq \emptyset$ and it can be computed in polynomial time by an LP.

Effect of Special Nominations

Lemma

Let q^{nom} be a nomination, v an exit, u an entry node with $\bar{\pi} = \pi_u \geq \pi_v, v \in V$ (shift pressure if necessary), and $q^{\text{nom}}_{u,v}$ a special nomination. Then,

- the pressure drop from u to v in $q^{\text{nom}}_{u,v}$ is larger than in q^{nom} and
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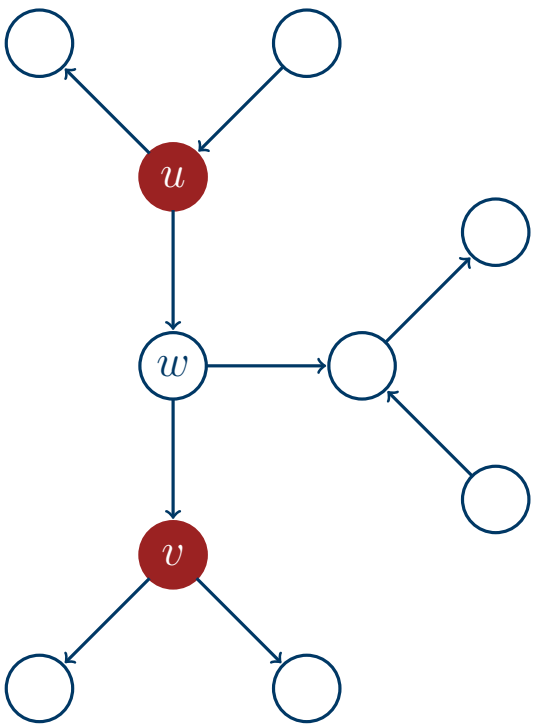

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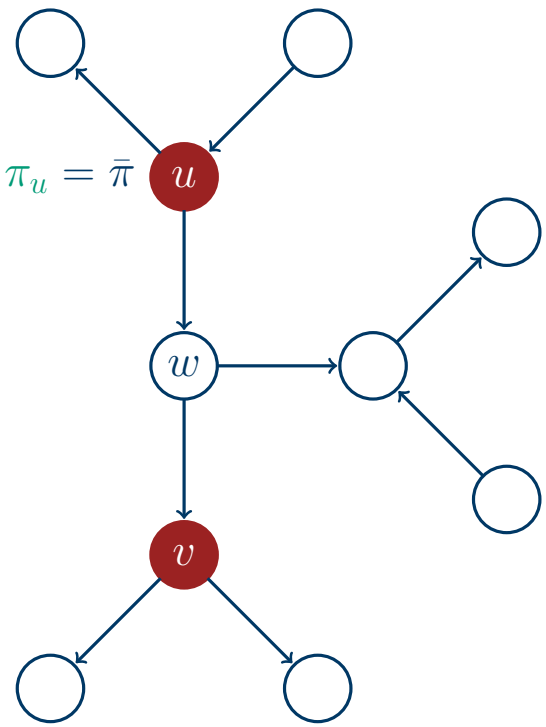

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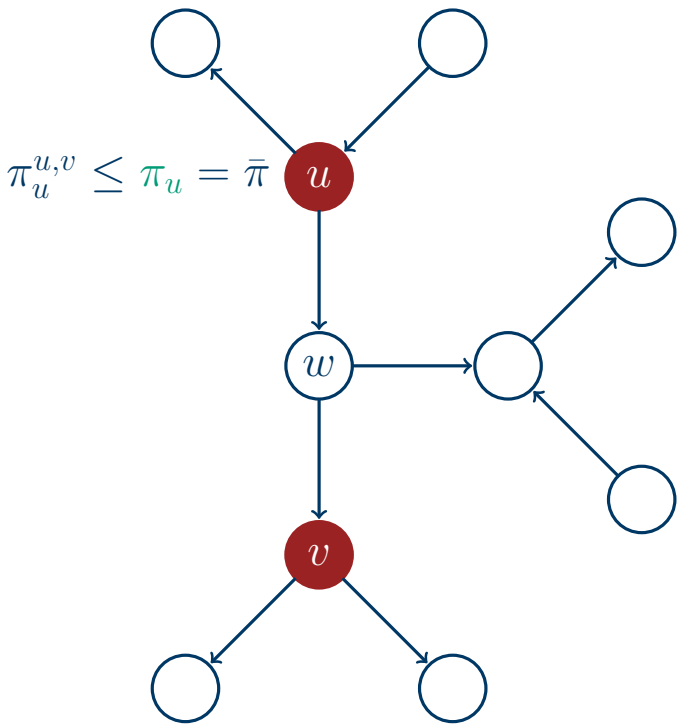

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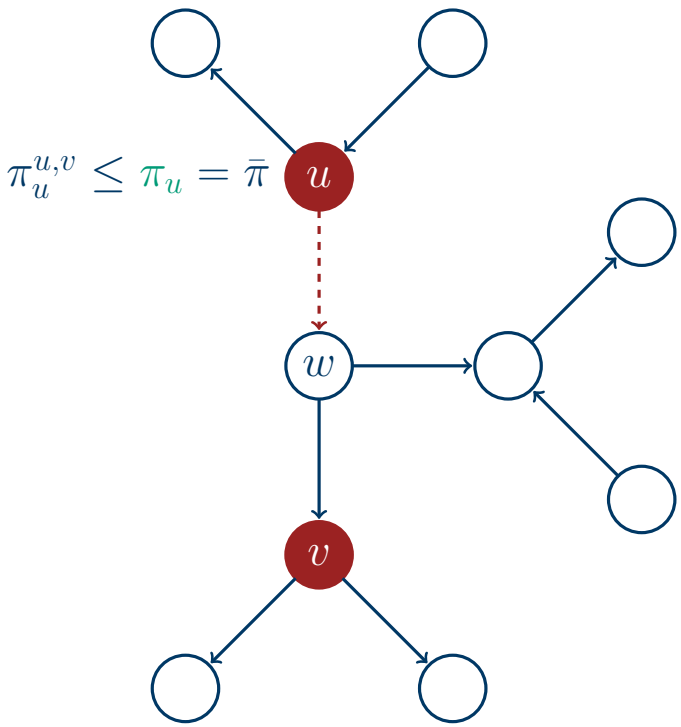

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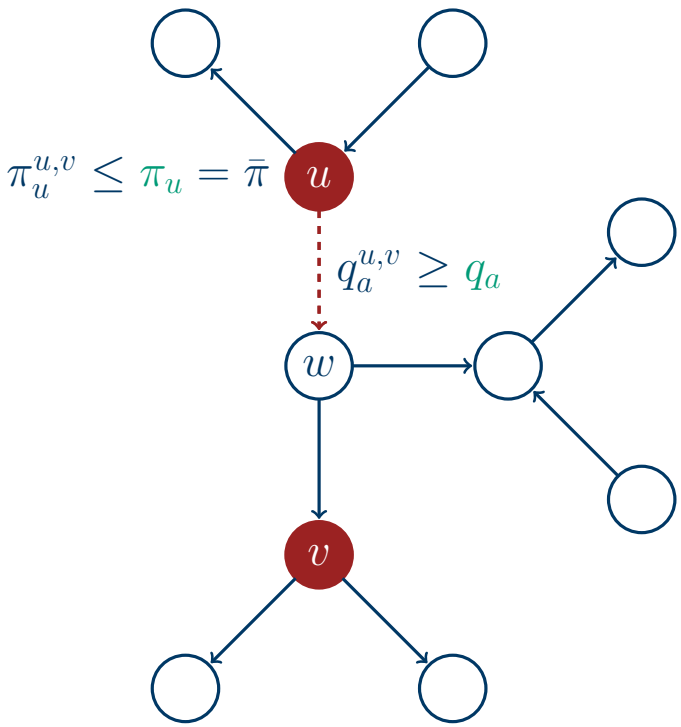

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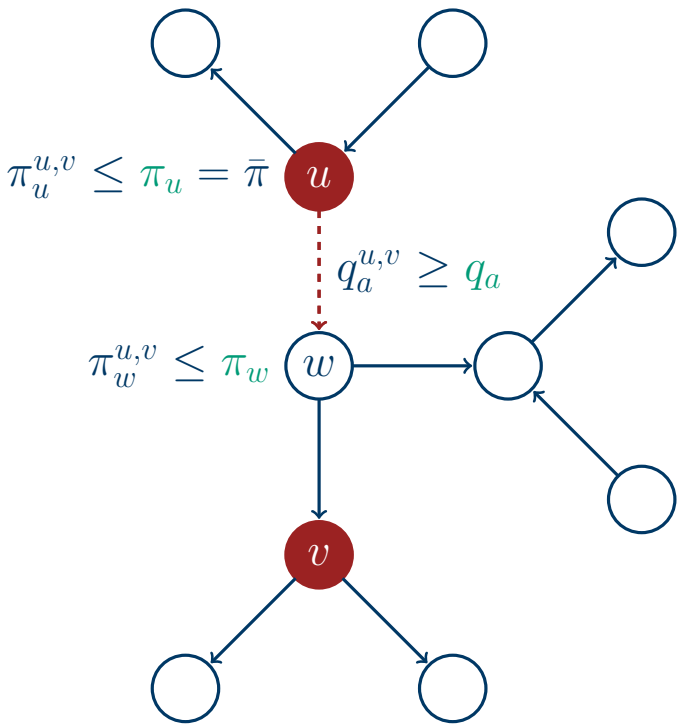

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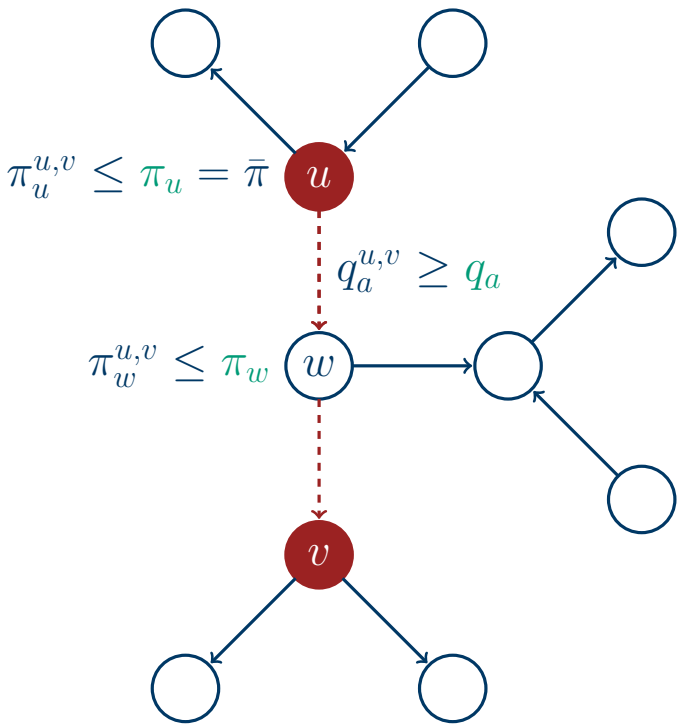

Effect of Special Nominations

Lemma

Let q^{nom} be a nomination, v an exit, u an entry node with $\bar{\pi} = \pi_u \geq \pi_v, v \in V$ (shift pressure if necessary), and $q^{\text{nom}}_{u,v}$ a special nomination. Then,

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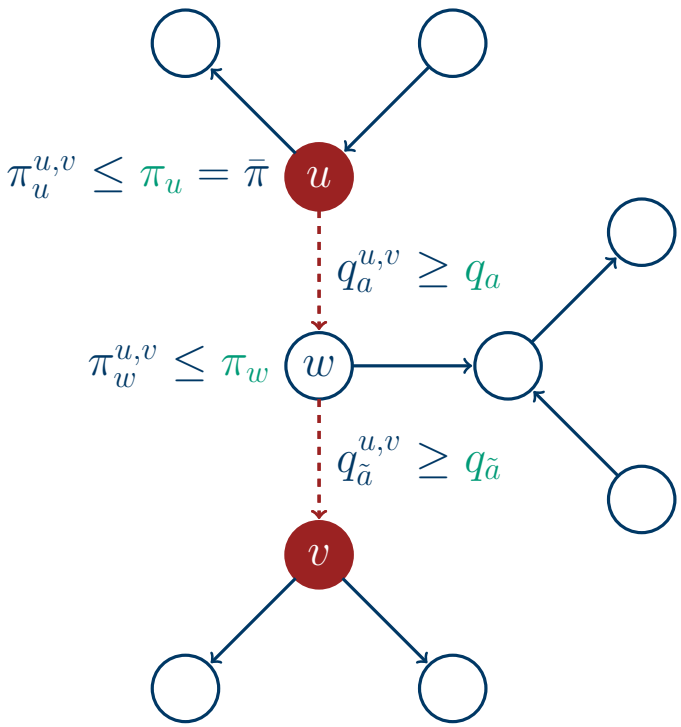

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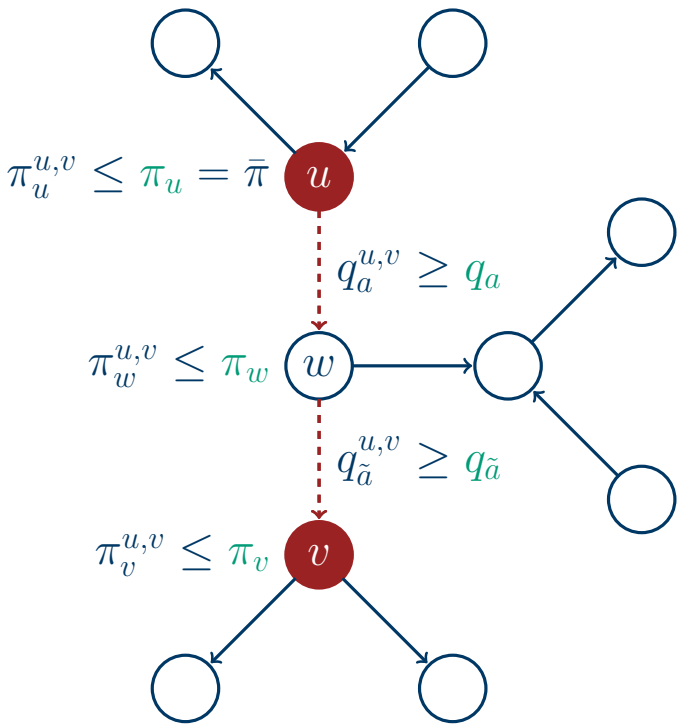

Diameter Selection of Hydrogen Networks

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$$B := \left\{ q^{\mathrm{nom}} \colon \sum_{v \in V} \sigma_u q_v^{\mathrm{nom}} = 0,\ 0 \leq q_v^{\mathrm{nom}} \leq q^{\mathrm{TC}},\ v \in V \right\}$$

Modeling

Nonlinear Model

Diameter Sizing Model

$$
\begin{aligned}
\min_{x,q,\pi} \quad & \sum_{a\in A}\sum_{d\in D_a} c_{a,d}\, x_{a,d} & & \\
\text{s.t.} \quad & \pi_{q_u^{\text{nom}}} - \pi_{q_v^{\text{nom}}} = \psi_a\left(\sum_{d\in D_a} d\, x_{a,d}, q_{q_a^{\text{nom}}}\right), & & a=(u,v)\in A,\ q^{\text{nom}}\in B,\\
& \sum_{a\in\delta^{\text{out}}(v)} q_{q_a^{\text{nom}}} - \sum_{a\in\delta^{\text{in}}(v)} q_{q_a^{\text{nom}}} = q_v^{\text{nom}}, & & v\in V,\ q^{\text{nom}}\in B,\\
& \underline{\pi}_v \le \pi_{q_v^{\text{nom}}} \le \bar{\pi}_v, & & v\in V,\ q^{\text{nom}}\in B,\\
& \sum_{d\in D_a} x_{a,d} = 1, & & a\in A,\\
& x_{a,d}\in\{0,1\}, & & a\in A,\ d\in D_a.
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s.t. Check feasibility of $|V^2|$ many special scenarios,

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Computational Results

Details of Instance

Instance

- Realistic hydrogen network of Eastern Germany planned by Forschungszentrum Jülich (IEK-3)
- 1 Entry, 1 storage, and 745 exits
- 28 diameters in range 0.1063 – 1.536 m
- Upper pressure bound $\bar{\pi} = 95$ bar

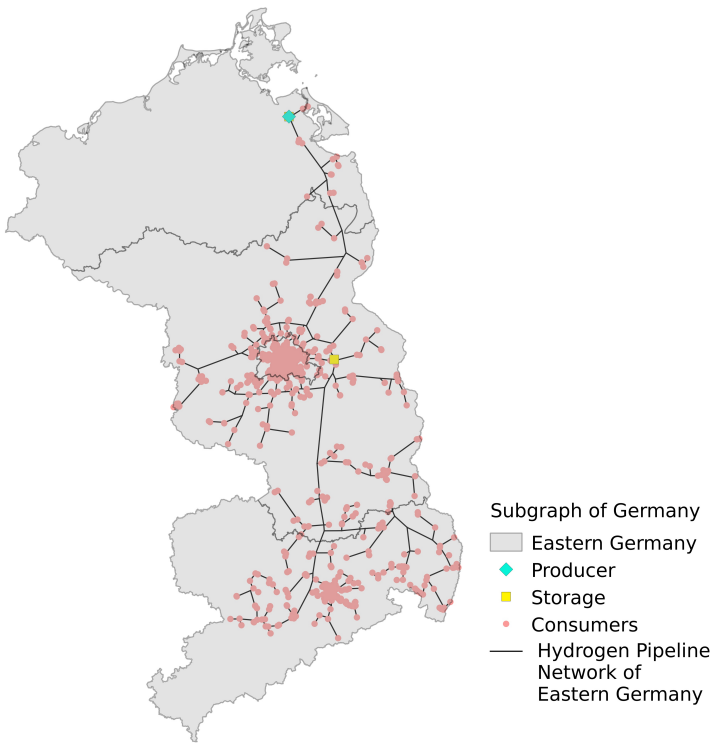

Figure: Hydrogen Network[2]

[2] Robinius, Schewe, Schmidt, Stolten, Thürauf, and Welder, "Robust Optimal Discrete Arc Sizing for Tree-Shaped Potential Networks".

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Results of Instance: Refueling Station without Local Onsite Storage

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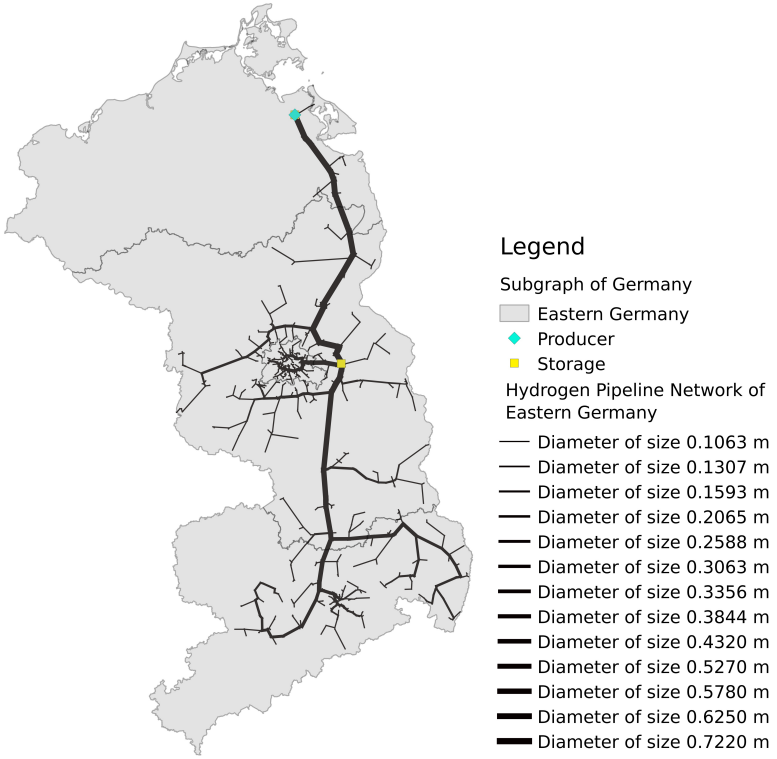

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Characterization of Technical Capacities

Theorem[3]

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What about cycles?

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Single Cycle Networks

Joint work with M. Labbé, F. Plein, M. Schmidt[4]

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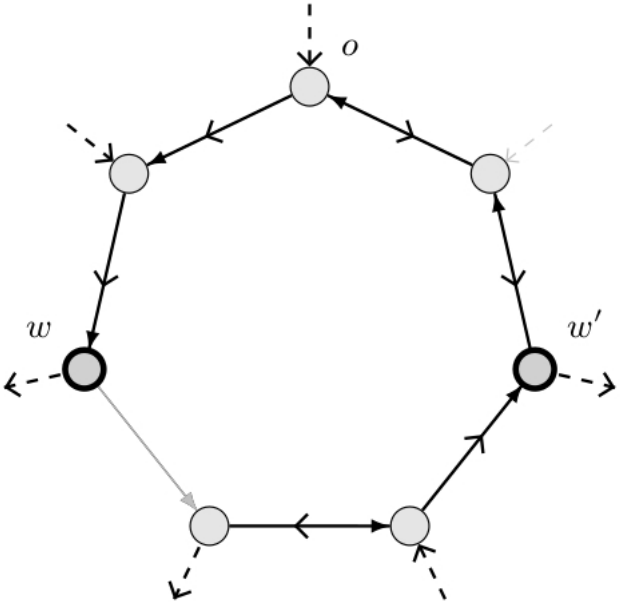

Figure: Different flow-meeting points w and w'.[4]

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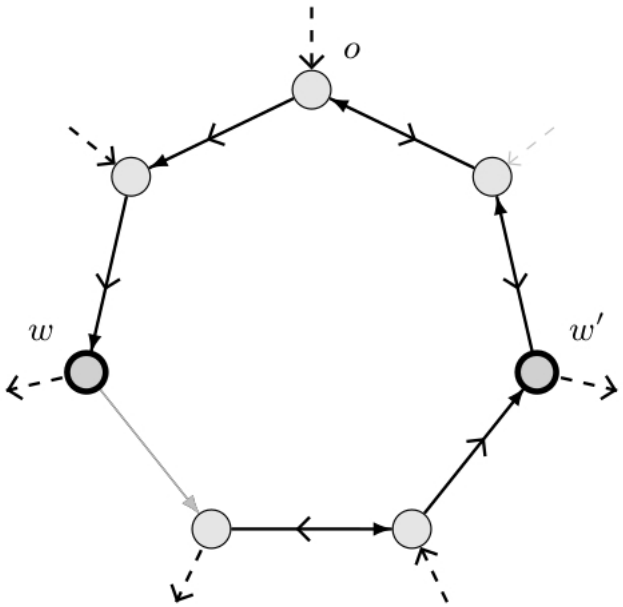

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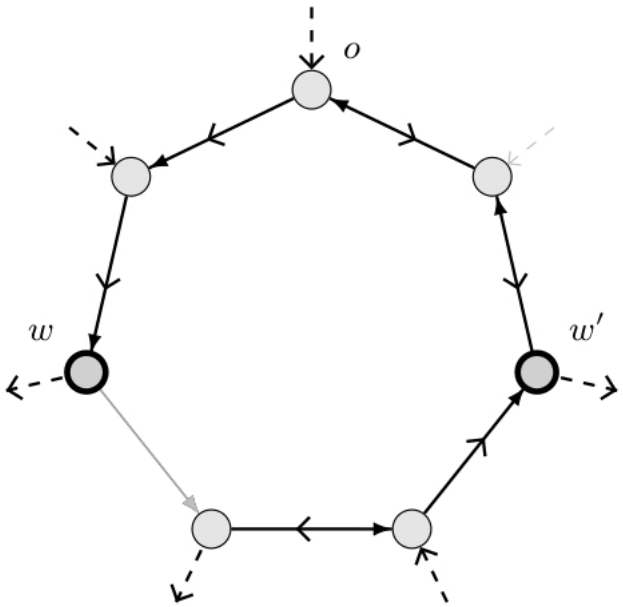
Why are cycles so bad?

No cyclic flows are possible

Different flow-meeting points

The potential drop in a cycle sums up to zero



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Theorem

It always exists an optimal solution with

- a single flow-meeting point
- a *certain* combinatorial structure and only polynomial many different possibilities for this structure exist.

Single Cycle Network

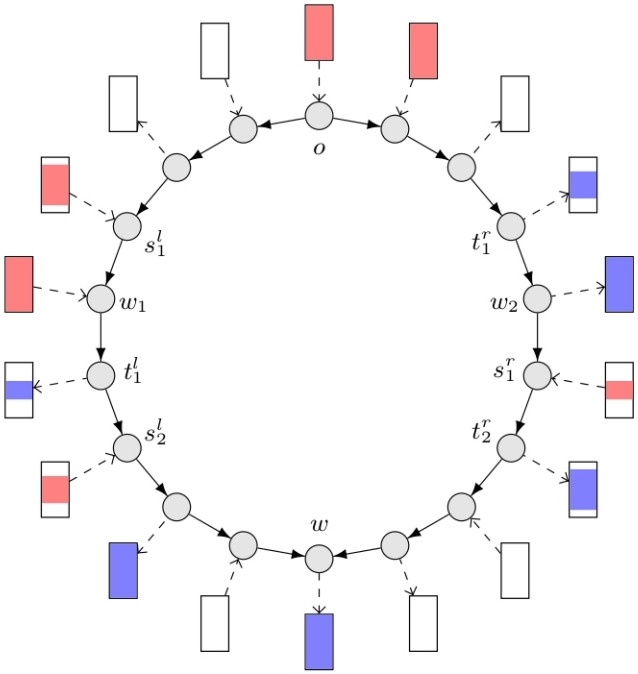

Figure: Properties of an Optimal Solution that Maximizes the Potential Difference between w_1 and w_2.[4]

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Single Cycle Network

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Result: Maximum potential difference between w_1 and w_2.
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We can decide the feasibility of technical capacities in $O((\log(V) + \tau) + |V^+|^5 |V^-|^4)$.

Complexity of Technical Capacities

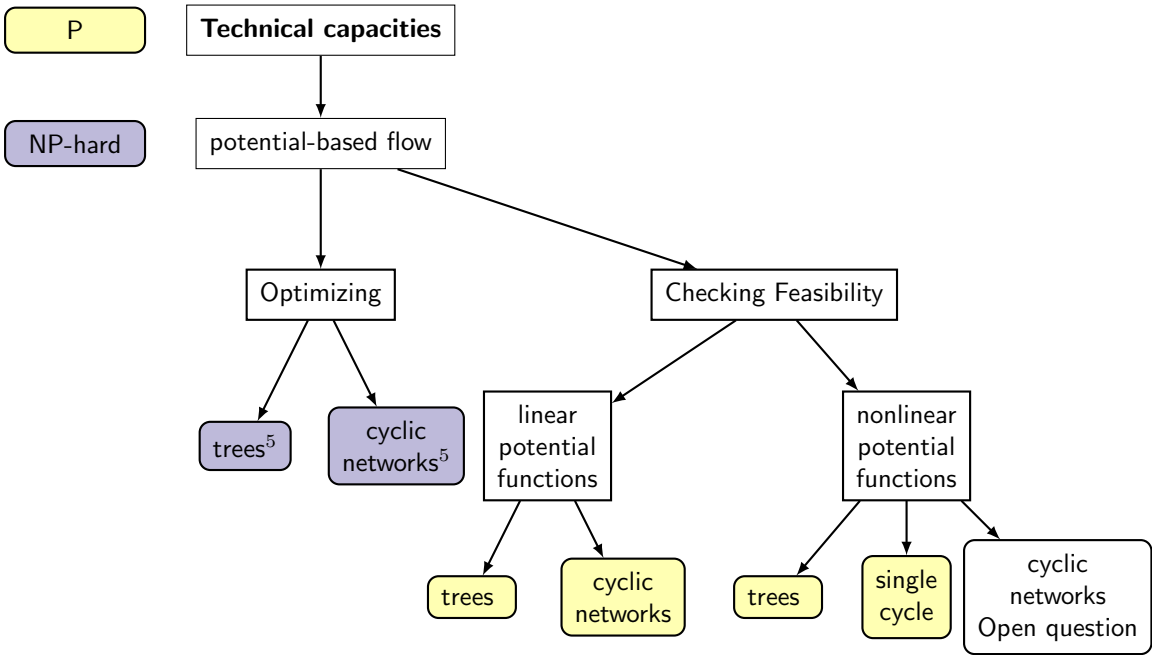

Figure: Overview of known complexity results for technical capacities and potential-based flows.

[5] Schewe, Schmidt, and Thürauf, *Computing Technical Capacities in the European Entry-Exit Gas Market is NP-Hard.*

Outlook

Future Research

Solve the nonlinear bilevel model for the Entry-Exit gas market system with nonlinear potential-based flows for trees.

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Finish my PhD

Journal Articles

- Martin Robinius, Lars Schewe, Martin Schmidt, Detlef Stolten, Johannes Thürauf, and Lara Welder
  Robust Optimal Discrete Arc Sizing for Tree-Shaped Potential Networks
  Published in Computational Optimization and Applications (2019):
  https://link.springer.com/article/10.1007/s10589-019-00085-x

- Markus Reuß, Lara Welder, Johannes Thürauf, Jochen Linßen, Thomas Grube, Lars Schewe, Martin Schmidt, Detlef Stolten, and Martin Robinius
  Modeling hydrogen networks for future energy systems: A comparison of linear and nonlinear approaches
  Published in International Journal of Hydrogen Energy (2019):
  https://www.sciencedirect.com/science/article/pii/S0360319919338625

Preprints

- Martine Labbé, Fränk Plein, Martin Schmidt, and Johannes Thürauf
  Deciding Feasibility of a Booking in the European Gas Market on a Cycle is in P
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Thank you for your attention