# Stochastic AC Optimal Power Flow (OPF): A Data-Driven Approach

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# Context and personal background

- PhD student in OR and Energy at CORE/LIDAM, UCLouvain since 2016 (https://uclouvain.be/fr/node/4474).
- Under the supervision of Anthony Papavasiliou (https://perso.uclouvain.be/anthony.papavasiliou).
- Part of the UCLouvain Engie Chair (http://uclengiechair.be/).

- Work done during summer 2019 at LANL, Theory Division. Group: Advanced Network Science Initiative (https://lanl-ansi.github.io/).
- Paper to appear in PSCC2020: https://arxiv.org/abs/1910.09144.

# Motivation

The increase in renewable generation and load flexibility comes with new challenges.

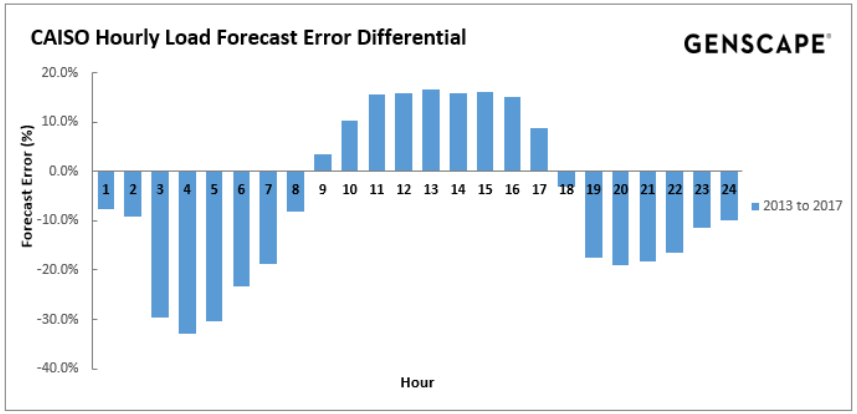
Intelligencer

BLACKOUT | UPDATED JULY 14, 2019

Power Back On After Blackout Strikes Much of Manhattan

*By Margaret Hartmann and Chas Danner*

Seen from the air, a large section of Manhattan's Upper West Side and Midtown neighborhoods sit coated in darkness during a partial blackout on July 13, 2019. Photo: Scott Heins/Getty Images




**Source:** https://www.genscape.com/blog

**→ Need for more reliable decisions.**

# Research question

Lot of historical data collected by power grid operators for the same static power network.

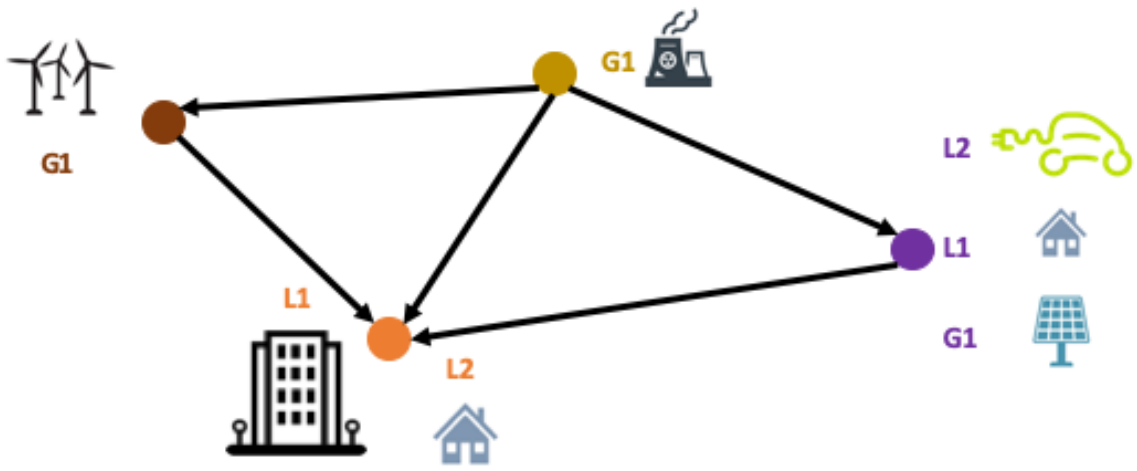
**How can historical data and network information be used efficiently to ensure reliable decision making on the grid?**

# Agenda

1. Introduction
2. Problem Formulation
3. A Data-Driven Scenario-Based Approach
4. Numerical Experiment
5. Conclusion & Future Work

# Optimal Power Flow

Goal of the problem: find the cheapest way to generate enough power to satisfy the demand without violating technical constraints.




Sets:

- Buses $\mathscr{B}$,
- Lines $\mathscr{L} = \mathscr{L}^f \bigcup \mathscr{L}^r$,
- Generators $\mathscr{G}$.

Variables:

- $p/q$ real/reactive generation,
- $f^p/f^q$ real/reactive power flow,
- $v/\theta$ voltage magnitude/angle.

# OPF and Power Flow (PF) Recourse

## Deterministic OPF

$$\min \quad \sum_{g \in \mathcal{G}} c(p_g) \tag{1}$$

## PF Recourse

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$$s.t. \quad \sum_{(i,j) \in \mathscr{L}} f_{ij}^p = \sum_{g \in \mathscr{G}_i} p_g - P_i - G_i^s v_i^2 \quad \forall i \in \mathscr{B} \tag{2}$$

$$\sum_{(i,j) \in \mathscr{L}} f_{ij}^q = \sum_{g \in \mathscr{G}_i} q_g - Q_i + B_i^s v_i^2 \quad \forall i \in \mathscr{B} \tag{3}$$

## PF Recourse

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$$f_{ij}^p = G_i v_i^2 - G_{ij} v_i v_j \cos(\theta_i - \theta_j) - B_{ij} v_i v_j \sin(\theta_i - \theta_j) \quad \forall (i,j) \in \mathscr{L} \tag{4}$$

$$f_{ij}^q = -B_i v_i^2 + B_{ij} v_i v_j \cos(\theta_i - \theta_j) - G_{ij} v_i v_j \sin(\theta_i - \theta_j) \quad \forall (i,j) \in \mathscr{L} \tag{5}$$

$$(f_{ij}^p)^2 + (f_{ij}^q)^2 \leq S_{ij}^2 \quad \forall (i,j) \in \mathscr{L} \tag{6}$$

## PF Recourse

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$$\underline{\theta_{ij}} \leq \theta_i - \theta_j \leq \overline{\theta_{ij}} \quad \forall (i,j) \in \mathscr{L}^f \tag{7}$$

$$\underline{p} \leq p \leq \overline{p}, \ \underline{q} \leq q \leq \overline{q}, \ \underline{v} \leq v \leq \overline{v} \tag{8}$$

**→ Non-linear, non convex optimization problem.**

## PF Recourse

# OPF and Power Flow (PF) Recourse

## Deterministic OPF

$$\min \quad \sum_{g \in \mathscr{G}} c(p_g) \tag{1}$$

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**→ Non-linear, non convex optimization problem.**

## PF Recourse

1. Fix $(p, v)$ for PV buses .
2. Find $(p, q, f^p, f^q, v, \theta)$ by solving (2)-(5).

**→ System of non-linear equalities (easy to solve).**

# OPF and Power Flow (PF) Recourse

Ideally: **find ($p^0$, $v^0$) and an 'adjustment policy' able to react in case of perturbations**.

Is this feature possible to ensure? If so, how?

We suggest:

- a formulation of Stochastic AC OPF (SACOPF).
- attacking the problem with a practical iterative approach.

# Stochastic AC OPF

We'll only assume load disturbances.

$\Omega$ denotes the uncertainty set. For $\omega \in \Omega$, the feasible set for OPF is:

$$
\begin{aligned}
\Gamma_{\text{OPF}}(\omega) \quad &= \{(p, q, f^p, f^q, v, \theta) \text{ satisfying} \\
&\sum_{(i,j)\in\mathscr{L}} f^p_{ij} = \sum_{g\in\mathscr{G}_i} p_g - (P_i + \mu_i^{\omega,p}) - G_i^s v_i^2 \quad \forall i \in \mathscr{B}, \\
&\sum_{(i,j)\in\mathscr{L}} f^q_{ij} = \sum_{g\in\mathscr{G}_i} q_g - (Q_i + \mu_i^{\omega,q}) + B_i^s v_i^2 \quad \forall i \in \mathscr{B}, \\
&\text{and } (4) - (8)\}
\end{aligned}
$$

# Stochastic AC OPF

Since $(p, v)$ need to be used for recourse, we suggest the following formulation:

$$(p^0(\Omega), v^0(\Omega)) = \arg\min \sum_{g \in \mathscr{G}} c_g(p_g^0) \tag{9}$$

$$s.t.\ (p^\omega, q^\omega, f^{p,\omega}, f^{q,\omega}, v^\omega, \theta^\omega) \in \Gamma_{\text{OPF}}(\omega) \qquad \forall \omega \in \Omega \tag{10}$$

$$p^\omega = p^0 + \left( \sum_{i \in \mathscr{B}} \mu_i^{p,\omega} \right) \alpha \qquad \forall \omega \in \Omega \tag{11}$$

$$v^\omega = v^0 \qquad \forall \omega \in \Omega \tag{12}$$

(11) and (12) define the adjustment policy.
Note that $\alpha$ is a parameter, $\alpha_g \approx \frac{1}{|\mathscr{G}|} \quad \forall g \in \mathscr{G}$.

# How to tackle SACOPF?

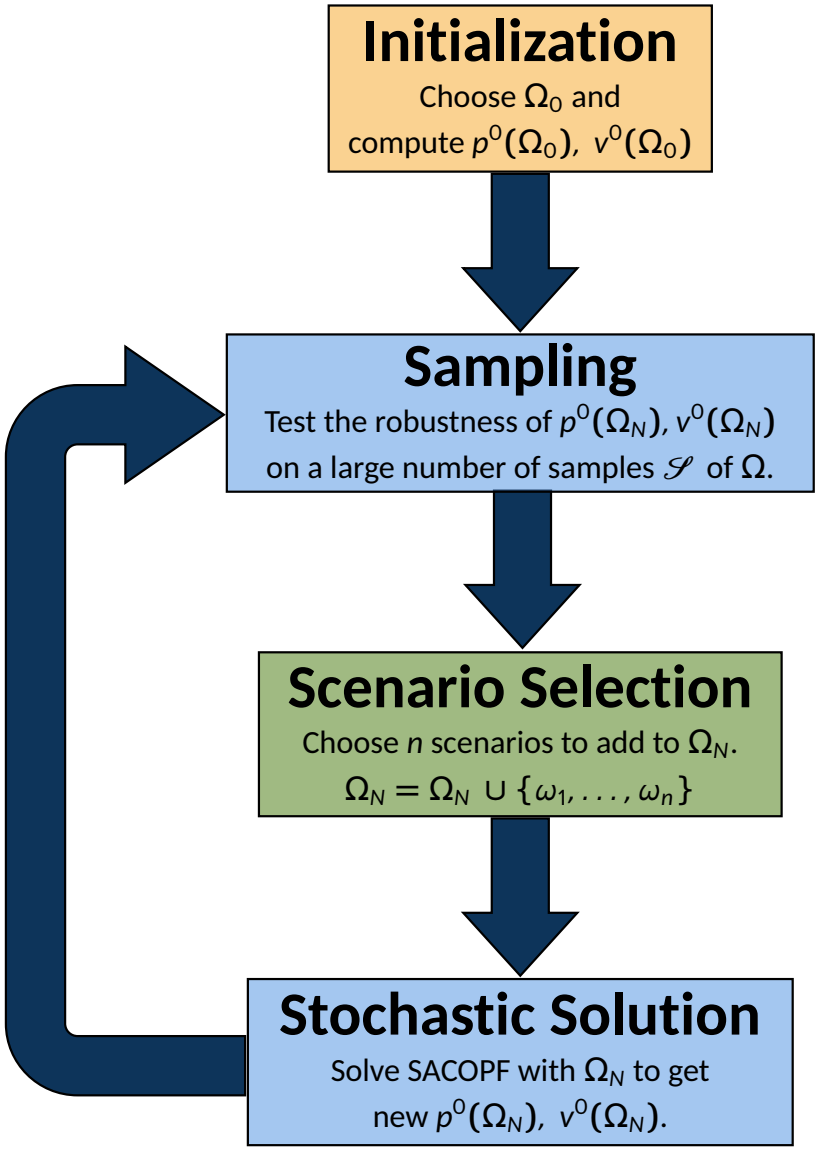
One main issue concerning $\Omega$:

- finite but huge if based on historical data.
- inifite if based on a probability distribution.

The idea is to **intelligently reduce $\Omega$ to $\Omega_N = \{\omega_1, \ldots, \omega_N\}$**, with $N$ small enough, and **compute $(p^0(\Omega_N), v^0(\Omega_N))$ in a way that ensures feasibility for all (or most of) $\omega \in \Omega$.**

# Agenda

# General Idea



# Toy Example

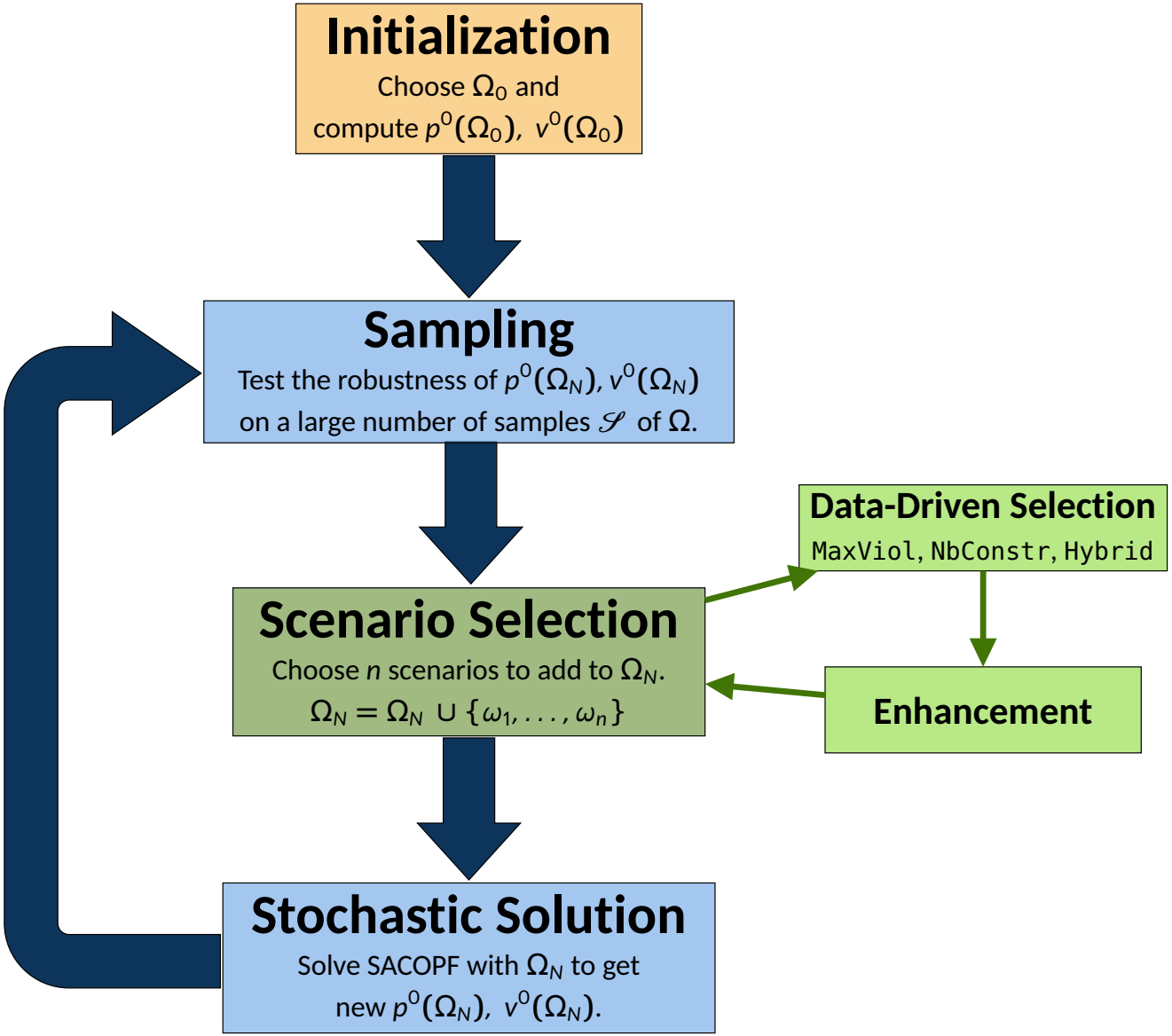
One simple way to apply the approach:

- $\Omega_0 = \{\omega_0\}$ where $\omega_0 = (\mu^p = 0, \mu^q = 0)$.
- Add $n$ random scenarios to $\Omega_0$.

Test case `73_ieee`: 73 bus-system, **51 loads**. We assume **max/min +/- 3% uniform perturbation of each load**.

| n | Infeasibility PF Recourse |
|---|---|
| 0 | **1,000/1,000** |
| 9 | **595/1,000** |
| 19 | **250/1,000** |
| 29 | **323/1,000** |
| 39 | **80/1,000** |
| 49 | **122/1,000** |

# Practical Approach



# Data-Driven Selection

We should use sampling information to choose the scenarios to add to $\Omega_N$.

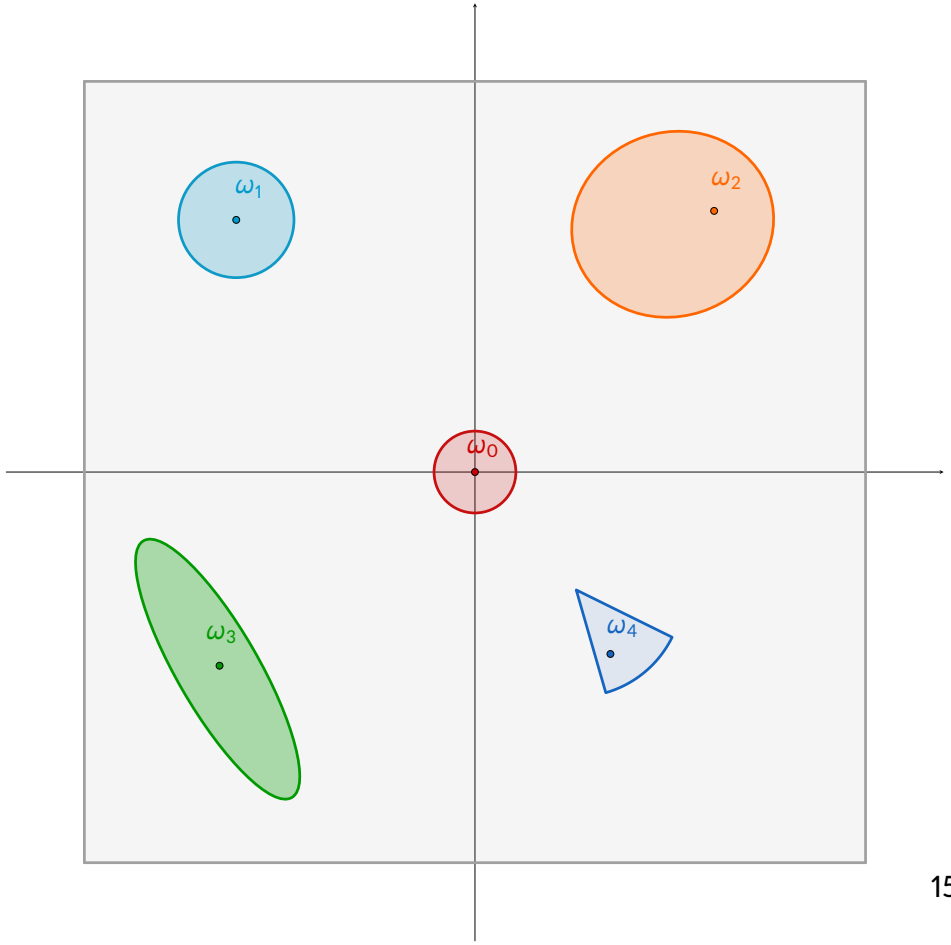
**Sampling**

Test the robustness of $p^0(\Omega_N)$, $v^0(\Omega_N)$ on a large number of samples $\mathscr{S}$ of $\Omega$.

**Scenario Selection**

Choose $n$ scenarios to add to $\Omega_N$.

$\Omega_N = \Omega_N \cup \{\omega_1, \ldots, \omega_n\}$



# Data-Driven Selection

How to select 'bad' scenarios?

*Example: we want to add 3 of these samples to $\Omega_N$.*

- Sample $s_1$. Constraints violated: `[QgUp10,FlowLim3,VDown12]`.
  Max violation: **`7.5%`**.
- Sample $s_2$: Constraints violated: `[QgUp10,VDown12]`.
  Max violation: **`5.0%`**.
- Sample $s_3$. Constraint violated: `[QgUp10]`.
  Max violation: **`15.0%`**.
- Sample $s_4$. Constraints violated: `[QgUp10,FlowLim3,VDown12]`.
  Max violation: **`2.2%`**.
- Sample $s_5$. Constraint violated: `[QgDown11]`.
  Max violation: **`9.0%`**.

# Data-Driven Selection

How to select 'bad' scenarios? Max Viol.

*Example: we want to add 3 of these samples to $\Omega_N$.*

- Sample $s_1$. Constraints violated: `[QgUp10,FlowLim3,VDown12]`. 3
  Max violation: **`7.5%`**.
- Sample $s_2$: Constraints violated: `[QgUp10,VDown12]`.
  Max violation: **`5.0%`**.
- Sample $s_3$. Constraint violated: `[QgUp10]`. 1
  Max violation: **`15.0%`**.
- Sample $s_4$. Constraints violated: `[QgUp10,FlowLim3,VDown12]`.
  Max violation: **`2.2%`**.
- Sample $s_5$. Constraint violated: `[QgDown11]`. 2
  Max violation: **`9.0%`**.

# Data-Driven Selection

How to select 'bad' scenarios? Number of constraints.

*Example: we want to add 3 of these samples to $\Omega_N$.*

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  Max violation: **`7.5%`**.
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  Max violation: **`9.0%`**.

# Data-Driven Selection

How to select 'bad' scenarios? Hybrid: $\text{weight}_s = \frac{MV_s}{\max_{s' \in \mathcal{S}} MV_{s'}} + \frac{NbC_s}{\max_{s' \in \mathcal{S}} NbC_{s'}}$

*Example: we want to add 3 of these samples to $\Omega_N$.*

- Sample $s_1$. Constraints violated: [QgUp10,FlowLim3,VDown12]. 1
  Max violation: **7.5%** $\text{weight}_{s_1} = 1.5$.
- Sample $s_2$: Constraints violated: [QgUp10,VDown12]. 3
  Max violation: **5.0%** $\text{weight}_{s_2} = 1$.
- Sample $s_3$. Constraint violated: [QgUp10]. 2
  Max violation: **15.0%** $\text{weight}_{s_3} = 1.33$.
- Sample $s_4$. Constraints violated: [QgUp10,FlowLim3,VDown12].
  Max violation: **2.2%** $\text{weight}_{s_4} = 1.15$.
- Sample $s_5$. Constraint violated: [QgDown11].
  Max violation: **9.0%** $\text{weight}_{s_5} = 0.93$.

# Toy Example

`73_ieee`: max/min +/- 3% uniform perturbation of each load.

- $\Omega_0 = \{\omega_0\}$ where $\omega_0 = (\mu^p = 0, \mu^q = 0)$.
- **Add $n = 5$ scenarios from the 1,000 samples to $\Omega_N$ using `MaxViol`, `NbConstr` or `Hybrid` selection .**

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| 0 | **1,000/1,000** |
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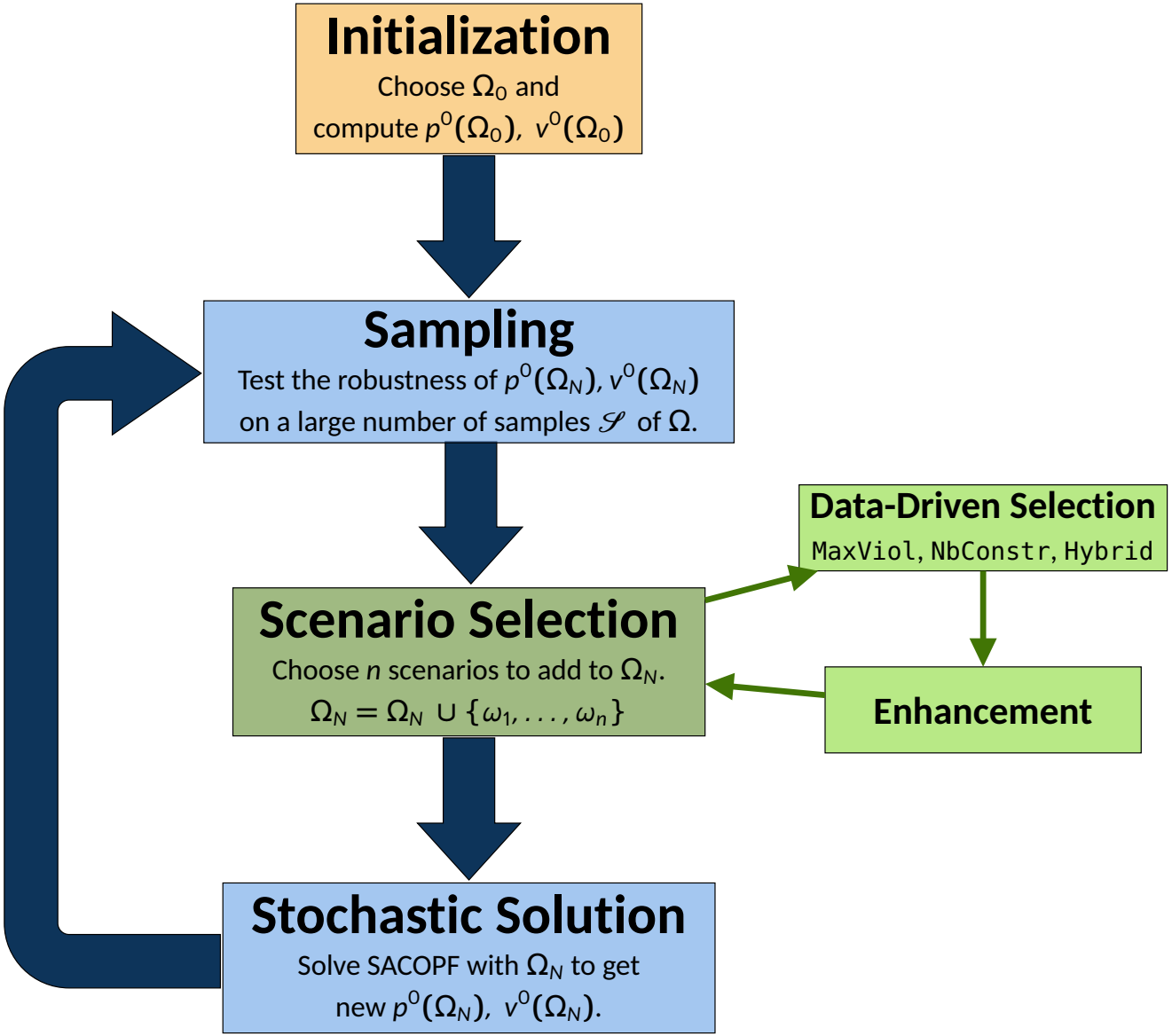
| Scen. Selec. | # It | $\lvert\Omega_N\rvert$ | PF Recourse |
|---|---|---|---|
| `MaxViol` | 5 | 20 | **1/1,000** |
| `NbConstr` | 7 | 28 | **0/1,000** |
| `Hybrid` | 8 | 29 | **0/1,000** |

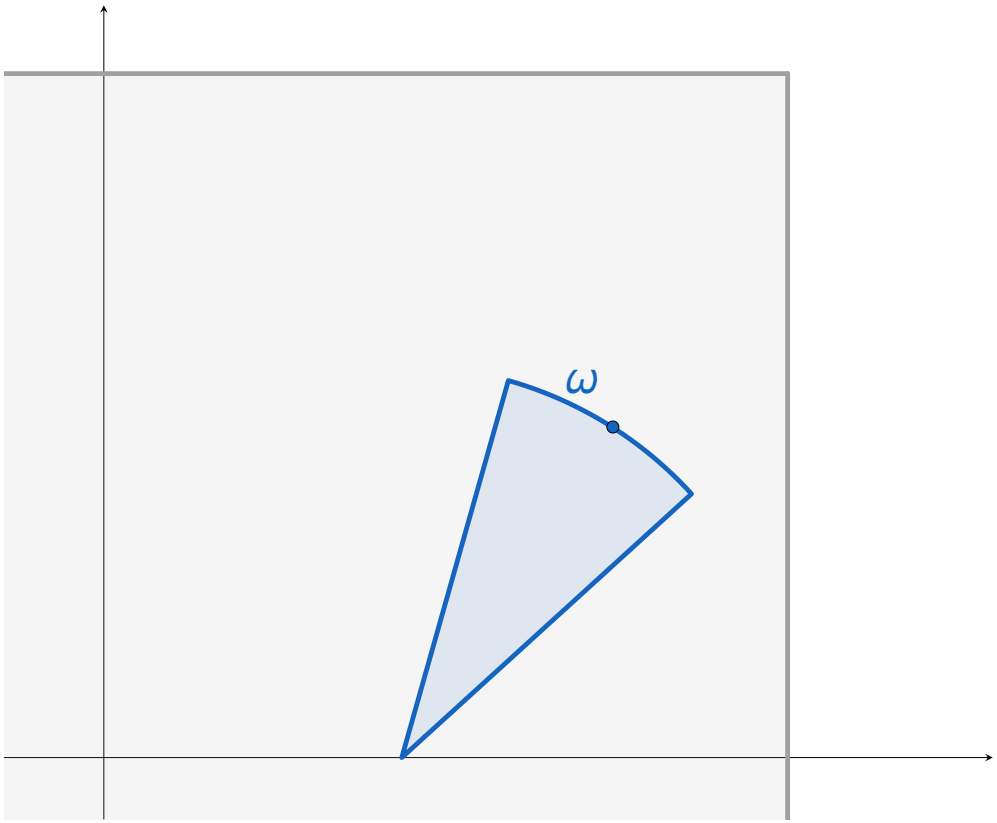
**Still, $\Omega_N$ might be too large at the end of the iterations, especially for this small test case.**

# Practical Approach



# Enhancement

After selecting 'bad' scenarios, would it be possible to make them capture 'more scenarios'?



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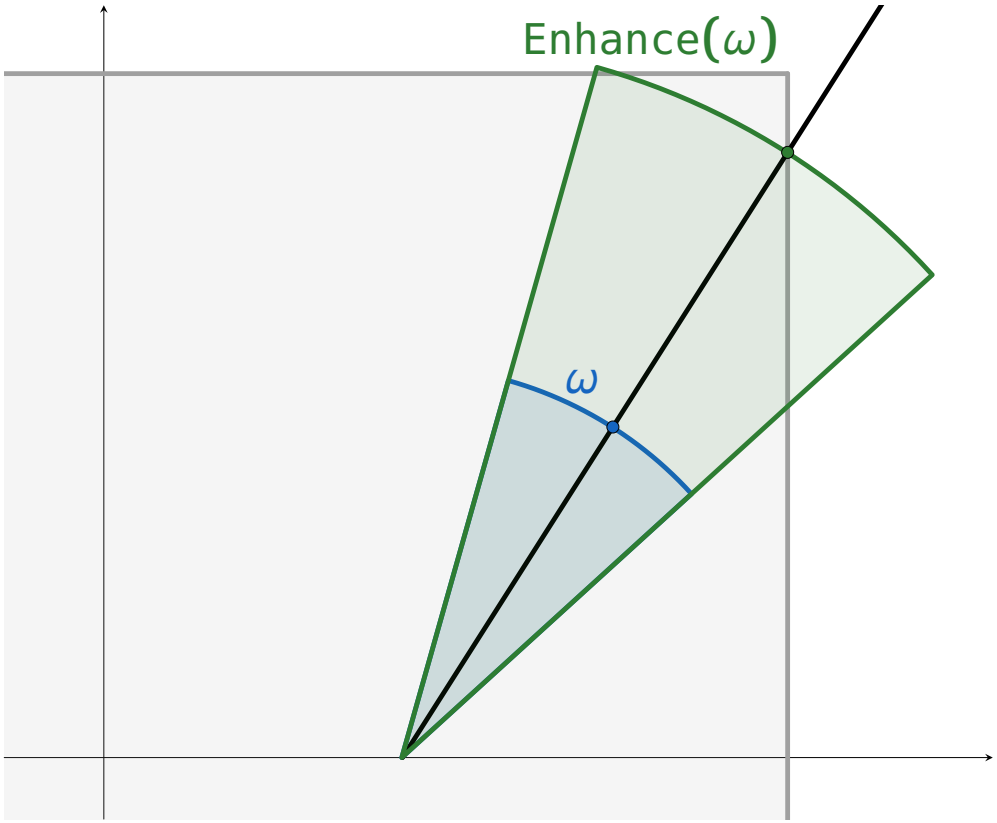

# Enhance operation

Let $\omega$ be a scenario to be added to $\Omega_N$.

$\mathscr{S}_c$: set of sampled scenarios violating constraint $c \in \mathscr{C}$.

$\mathscr{C}_\omega$: set of constraints violated by $\omega$.

$y_c^s$: violation of constraints $c$ by sampled scenario $s$.

$$\rightarrow \forall c \in \mathscr{C}_\omega,\ d^c = \arg\min_d \sum_{s \in \mathscr{S}_c} \left( y_c^s - (d_0 + \textstyle\sum_{i \in \mathscr{B}} d_i \mu_i^s) \right)^2 + \lambda |d|_1$$

$\rightarrow$ Deduce $d^\omega$ by gathering non zero directions of $d^c,\ c \in \mathscr{C}_\omega$.

$\rightarrow$ $\forall i \in \mathscr{B}$, depending on $\text{sign}(d_i^\omega)$,

$$\mu_i^\omega = \text{Enhance}(\mu_i^\omega) = \overline{\mu}_i, \underline{\mu}_i \text{ or } \mu_i^\omega$$

.

# Toy Example

`73_ieee`: max/min +/- 3% uniform perturbation of each load.

- $\Omega_0 = \{\omega_0\}$ where $\omega_0 = (\mu^p = 0, \mu^q = 0)$.
- $n = 5$, $|\mathscr{S}| = 1,000$ using `MaxViol`, `NbConstr` or `Hybrid` selection and **applying Enhance**.

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|---|---|---|---|
| `MaxViol` | 5 | 20 | **1/1,000** |
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| Scen. Selec. | # It | $\lvert\Omega_N\rvert$ | PF Recourse |
|---|---|---|---|
| `MaxViol` | 1 | 6 | **0/1,000** |
| `NbConstr` | 2 | 11 | **0/1,000** |
| `Hybrid` | 2 | 11 | **0/1,000** |

# Agenda

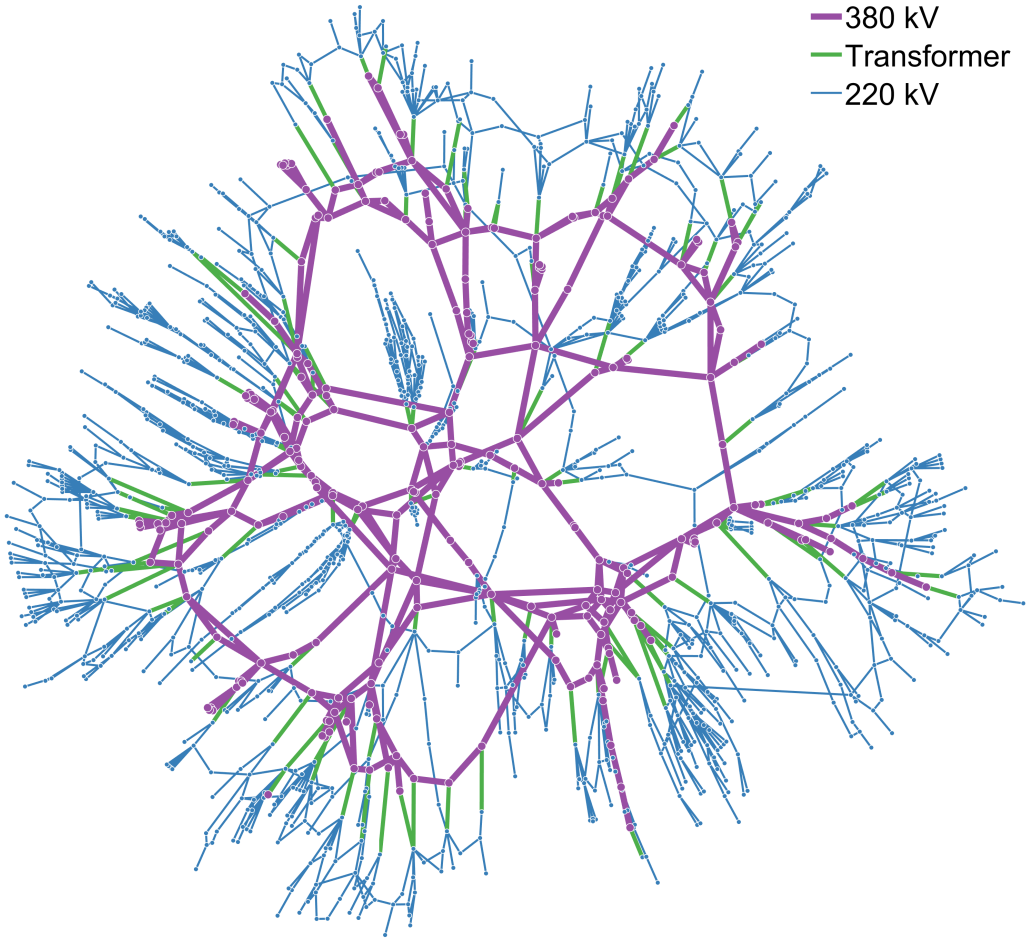
# Numerical experiment

The method works on small test cases. For these 3 test cases, we apply +/- 3 % at each load.

| Test case | # Loads | Scen. Selec. | # It | $\lvert\Omega_N\rvert$ | PF Recourse |
|---|---|---|---|---|---|
| `24_ieee` | 17 | `MaxViol` | 3 | 7 | **0/1,000** |
| `73_ieee` | 51 | `MaxViol` | 1 | 6 | **0/1,000** |
| `118_ieee` | 99 | `MaxViol` | 3 | 14 | **0/1,000** |

# Numerical experiment

One large test case: `1354_pegase`. **+/- 2% for loads located at leaf buses: 211 uncertain loads**.



# 1354_pegase

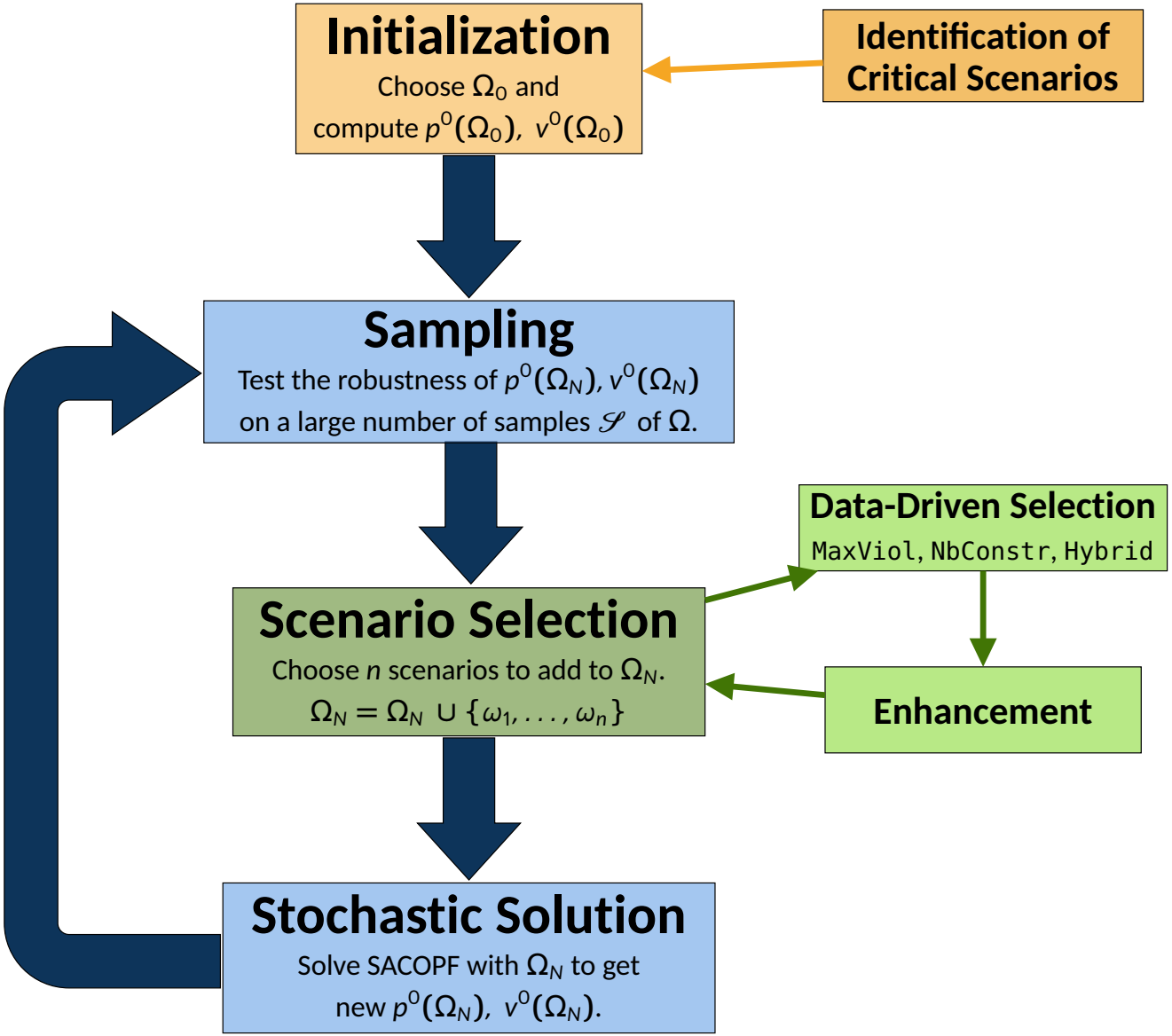
How does the method scale up?

| Scen. Selec. | # It | $\lvert\Omega_N\rvert$ | PF Recourse | Max. Viol. | Exp. Viol. |
|---|---|---|---|---|---|
| `MaxViol` | 6 | 31 | **10/1,000** | 0.17% | 0.06% |

$\longrightarrow$ Very promising results!

**Would it be possible to get better if we choose a better $\Omega_0$?**

# Practical Approach



# Identification of Critical Scenarios

At the moment, we only initialize $\Omega_0 := \{\omega_0\}$.
Could we find a better way to initialize the algorithm?

- If historical data is available, an operator would probably have an idea of what critical scenarios could be.

# Identification of Critical Scenarios

At the moment, we only initialize $\Omega_0 := \{\omega_0\}$.

Could we find a better way to initialize the algorithm?

- Otherwise, we suggest to detect critical scenarios in the following way:
  1. Take the **deterministic solution** and consider $\mu$ **as a variable**.
  2. Change the objective: **Maximize violation of a certain constraint**.
  3. For each constraint violated $\longrightarrow$ a **critical scenario**.
  4. If necessary, **cluster the critical scenarios** to reduce the size of potential $\Omega_0$.

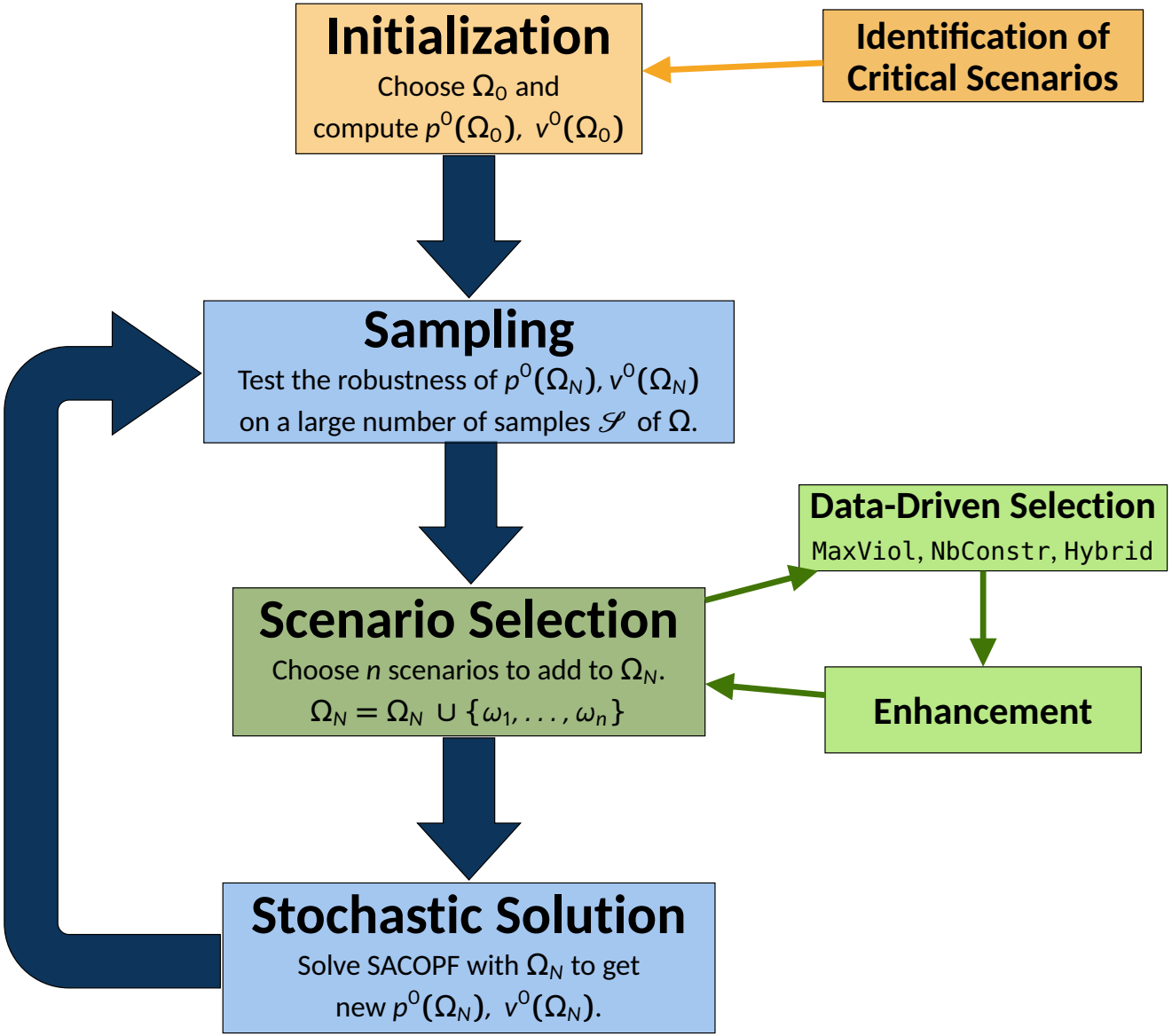
# 1354_pegase

| Scen. Selec. | # It | $\lvert\Omega_N\rvert$ | PF Recourse |
|---|---|---|---|
| MaxViol | 6 | 31 | **10/1,000** |

Applying this to 1354_pegase, we obtained **392 critical scenarios and reduced it to 11 scenarios using K-means clustering in order to get $\Omega_0$.**

| Scen. Selec. | # It | $\lvert\Omega_N\rvert$ | PF Recourse |
|---|---|---|---|
| Hybrid | 3 | 25 | **0/1,000** |

# Practical Approach



# Conclusion & Future Work

- The formulation of the problem and the practical approach confirm that reliable decisions can be taken for solving AC-OPF.
- Numerical experiments seem promising:
  - `1354_pegase`: $|\Omega_N| = 25$ for 211 perturbed loads on a 1354 bus-system.
- Future work:
  - Parallelization: Initialization and Sampling.
  - Initial clustering could be improved.
  - More realistic uncertainty modeling.
  - Extend the approach to larger and more realistic test cases.

Thank you for your attention!

*Stochastic AC Optimal Power Flow:*
*A Data-Driven Approach*
https://arxiv.org/abs/1910.09144
To appear in PSCC2020

Ilyès Mezghani
https://sites.google.com/view/ilyesmezghani/home