Hall's Notes and Queries

NQ5

The volume of an out-of-true block

In WT3e[1], p33, note 12, Hoff and I wrote: 'Measuring the dimensions of the test specimen is a rapid and often excellent way to estimate the bulk volume V_b. However, it can be surprisingly difficult to cut an accurately rectangular block with flat faces, and if that is not achieved simple calculations of V_b may be in error. For such out-of-true specimens, there are more elaborate ways to calculate the volume by measuring the lengths of all edges and some face diagonals, but the extra measurements and calculations nullify the apparent simplicity of the approach.' Here we say a little more about this.

The widely-used ("standard") method to obtain the volume of a test block uses the Archimedes' weight. This method is not always convenient or available, and for many test specimens (blocks or cylindrical cores) direct measurement of the dimensions is a quicker and easier alternative. However experience shows that nominally rectangular blocks or prisms are often not cut accurately, and the blocks are not truly square. As a result the simple estimate of volume, V_{sim} = length $\times$ width $\times$ height, may overestimate the volume. A

[1] C Hall & W D Hoff, Water transport in brick, stone and concrete, third edition, CRC Press, 2021.

better estimate of the block volume can be made from the lengths of all twelve edges and some face diagonals. The calculation is based on the fact that any hexahedral block with six plane quadrilateral faces can be decomposed into five tetrahedra. This does not require the faces to be rectangles. The volume of each tetrahedron can be calculated from its edge lengths. The overall calculation of the block volume is rather elaborate, but once coded it can be applied easily.

Outline of calculation method. The block is treated as a cuboid or irregular hexahedron, and its volume is calculated from the lengths of the twelve edges, and of six diagonals (one diagonal of each face). It is assumed that all faces are flat. The cuboid is considered to to be made up of five irregular tetrahedra. One of these is constructed on four non-adjacent vertices of the cuboid, while the each of the other four tetrahedra contains one of the remaining corners of the cuboid. From the lengths of the edges and of the diagonals, the volume of each tetrahedron is calculated using Tartaglia's formula. Useful mathematical background is given by Wirth and Dreiding[2].

For the purposes of calculation it is essential to label the eight vertices of the the block in a systematic way. The indices shown in fig 1 are used here.

Code for calculation A simple code for this calculation is given below. The test data provided are micrometer measurements on a nominally rectangular brick prism used in the round-robin exercise reported by Feng et al[3]. Here the estimated volume from tetrahedral decomposition is 3.690×10^5 mm^3, while the simple volume from

[2] K Wirth & A S Dreiding (2009). Edge lengths determining tetrahedrons. Elem. Math., v64, 160–170.

[3] C Feng et al. (2020). Hygric properties of porous building materials (VI): A round robin campaign. Build. Environ., v185, 107242.

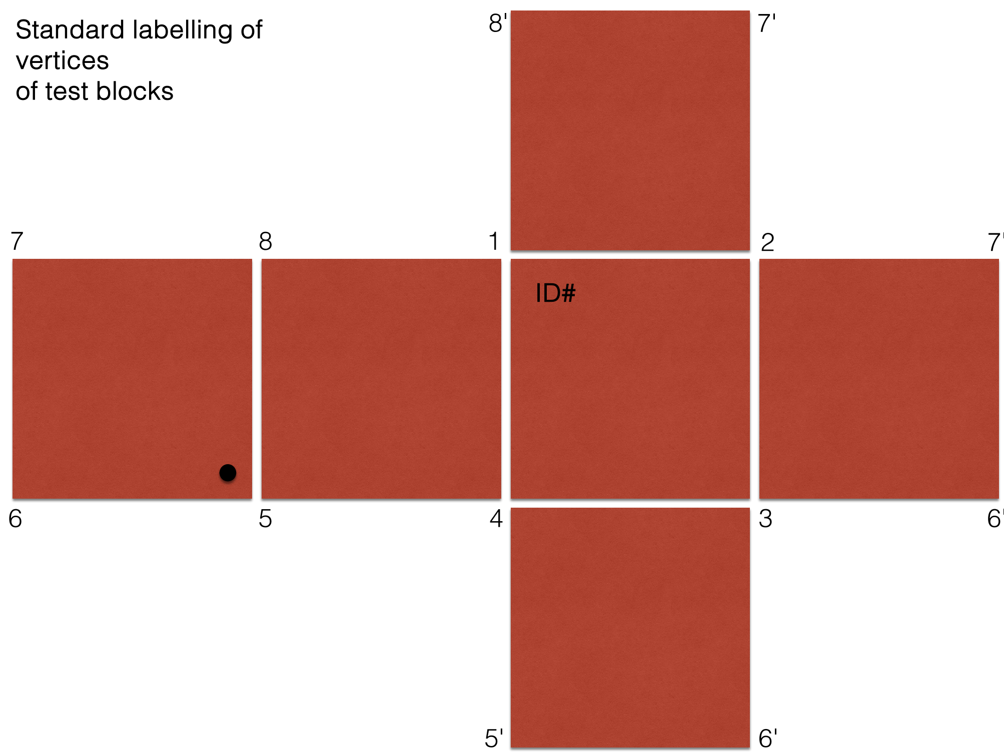

Figure 1: One face is chosen as the reference face and its corners labelled 1234 clockwise from the top left. The opposite face is similarly labelled 5678

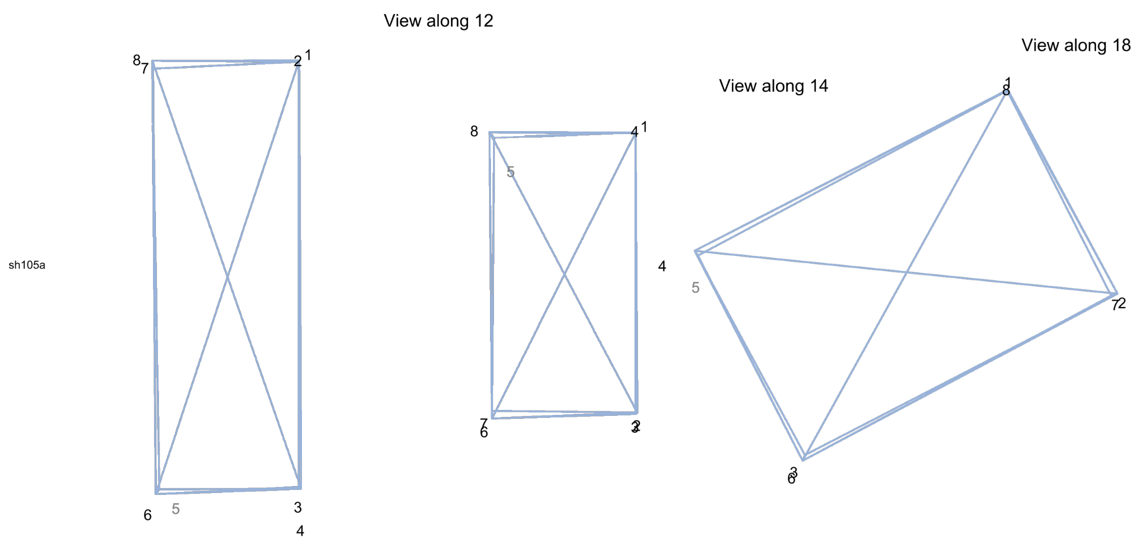

Figure 2: Views of the Robusta brick block reconstructed from edge lengths and face diagonals

mean edge lengths assuming the block is accurately rectangular is $3.731 \times 10^5\,\mathrm{mm}^3$.

A more elaborate code tests for planarity of faces and constructs the block geometry, fig 2.

The approach may have some merit in producing a small but useful improvement in accuracy in V_b, and in avoiding the laborious (and not necessarily accurate) Archimedes method. It does however require that the faces are flat or at least almost so.

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```
%CUBOIDVOLNQ5
%Matlab script

%11 Sept 2016
%14 Jul 2017 Minor mods
%28 Apr 2024 Tidied for NQ5

%Calculates volume of irregular cuboid
%Quadrilateral-faced hexahedron

%Irregular cuboid decomposed into 5 tetrahedra: one
%interior, and 4 exterior. In regular cuboid these have
%volumes 1/3 and 1/6 of total cuboid volume. Uses
%Tartaglia's formula for volume of tetrahedron in
%TETRAVOL2

%Data DD0 on edge and diagonal lengths usually in (mm)
%Using standard vertex numbering,
%DD0  block dimensions
%d12 d23 d34 d14 | d56 d67 d78 d58 |
%d36 d27 d18 d45 |
%d13 d15 d17 d35 d37 d57



%Test data: Robusta brick (C Feng et al., Build. Environ.
%v185, 107242)
```

```
%Rectangular prism, nominal 120 x 80 x 40 mm
%Measured dimensions (mm)
dd0=[78.09 118.64 78.24 118.78 78.07 118.68 77.92 118.88];
dd0=[dd0 40.0 40.76 40.77 39.43];
dd0=[dd0 140.86 125.12 87.33 86.36 123.57 140.84];



ddi=dd0([13:18]);  %Interior tetrahedron

dde2=dd0([1 2 10 13 15 17]); %Ext tetrahedra by vertex
dde4=dd0([4 3 12 13 14 16]);
dde6=dd0([9 5 6 16 17 18]);
dde8=dd0([11 8 7 14 15 18]);

Vi=tetravol2(ddi);   %Tetrahedron volumes
Ve1=tetravol2(dde2);
Ve2=tetravol2(dde4);
Ve3=tetravol2(dde6);
Ve4=tetravol2(dde8);

Vcub=Vi+Ve1+Ve2+Ve3+Ve4;   %Total cuboid volume


function vt=tetravol2(dd)

%Calculates volume of tetrahedron from Tartaglia formula

%DD six tetrahedron edge lengths
%Have regard to order: DD(1:3) have a common vertex,
%DD(4:5) have a common vertex; DD(6) is the remaining side
```

```
%AA is the Cayley-Menger determinant

AA=[[0 1 1 1 1];
    [1 0 dd(1)^2 dd(2)^2 dd(3)^2];
    [1 dd(1)^2 0 dd(4)^2 dd(5)^2];
    [1 dd(2)^2 dd(4)^2 0 dd(6)^2];
    [1 dd(3)^2 dd(5)^2 dd(6)^2 0]];

V4=sqrt(det(AA)/288);

vt=V4;
end

%[end cuboidvolNQ5.m]
```