Rotating Spirals in competition-diffusion systems

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We seek non trivial rigidly rotating solutions to time-dependent competition-diffusion systems of the type

$$\begin{cases} \partial_t u_i - \Delta u_i = f(u_i) - \beta u_i \sum_{j \neq i} a_{ij} u_j & \text{in } \Omega \times \mathbb{R}^+ \\ u_i(\mathbf{x}, 0) = u_{i,0}(\mathbf{x}) & \text{for } \mathbf{x} \in \Omega \end{cases}$$

Here Ω is a rotationally invariant planar set and $a_{ij} > 0$ for every i and j. We tackle the Neumann and the (also inhomogeneous) Dirichlet boundary conditions, as well as entire solutions in the plane. For the homogenous Neumann problem with logistic reactions we use a multi-parameter bifurcation argument to detect nontrivial solutions in the three-component case. Next, we let $\beta \to +\infty$ and we investigate the limit segregated problem for linear reactions $f(s) = \mu s$. For the Dirichlet problem of the singular limit problem, we give a complete characterization of the boundary traces φ_i $(i = 1, \dots, K)$ supporting spiraling waves, rotating with a given angular speed ω. As a byproduct of our analysis we detect explicit families of eternal, entire solutions of the pure heat equation, parameterized by $\omega \in \mathbb{R}$, which reduce to homogeneous harmonic polynomials for $\omega = 0$.

It is are joint works with Z. Lin, A. Salort, G. Verzini and A. Zilio

References

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